Package 'rSPDE'

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Type Package

Title Rational Approximations of Fractional Stochastic Partial Differential Equations

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Description Functions that compute rational approximations of fractional elliptic stochastic partial differential equations. The package also contains functions for common statistical usage of these approximations. The main reference for the methods is Bolin and Kirchner (2020) <doi:10.1080/10618600.2019.1665537>, which can be generated by the citation function in R.

Depends R (>= 3.5.0), Matrix

Imports stats, methods, utils

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`bru_mapper.inla_rspde` *rSPDE inlabru mapper*

Description

rSPDE inlabru mapper

Usage

```
bru_mapper.inla_rspde(model, ...)

ibm_n.bru_mapper_inla_rspde(mapper, ...)

ibm_values.bru_mapper_inla_rspde(mapper, ...)

ibm_amatrix.bru_mapper_inla_rspde(mapper, input, ...)

bru_get_mapper.inla_rspde(model, ...)
```

Arguments

`model`	An `inla_rspde` for which to construct or extract a mapper
`...`	Arguments passed on to other methods
`mapper`	A `bru_mapper.inla_rspde` object
`input`	The values for which to produce a mapping matrix

Examples

```
 #tryCatch version
tryCatch({
if (requireNamespace("INLA", quietly = TRUE) &&
     requireNamespace("inlabru", quietly = TRUE)){
library(INLA)
library(inlabru)

set.seed(123)
m <- 100
loc_2d_mesh <- matrix(runif(m * 2), m, 2)
mesh_2d <- inla.mesh.2d(
  loc = loc_2d_mesh,
  cutoff = 0.05,
  max.edge = c(0.1, 0.5)
)
sigma <- 0.01
range <- 0.2
nu <- 0.8
kappa <- sqrt(8 * nu) / range
op <- matern.operators(
```

```
  mesh = mesh_2d, nu = nu,
  kappa = kappa, sigma = sigma, m = 2
)
u <- simulate(op)
A <- inla.spde.make.A(
  mesh = mesh_2d,
  loc = loc_2d_mesh
)
sigma.e <- 0.1
y <- A %*% u + rnorm(m) * sigma.e
y <- as.vector(y)

data_df <- data.frame(y=y, x1 = loc_2d_mesh[,1],
                      x2 = loc_2d_mesh[,2])
coordinates(data_df) <- c("x1", "x2")
rspde_model <- rspde.matern(
  mesh = mesh_2d,
  nu_upper_bound = 2
)

# For inlabru version 2.5.3.9002 or above:
cmp <- y ~ Intercept(1) +
           field(coordinates, model = rspde_model)

#For inlabru version 2.5.3:
cmp <- y ~ Intercept(1) +
           field(coordinates, model = rspde_model,
             mapper = bru_mapper(rspde_model))

rspde_fit <- bru(cmp, data = data_df)
}
#stable.tryCatch
}, error = function(e){print("Could not run the example")})
```

`folded.matern.covariance.1d`

The 1d folded Matern covariance function

Description

`folded.matern.covariance.1d` evaluates the 1d folded Matern covariance function over an interval $[0, L]$.

Usage

```
folded.matern.covariance.1d(
  h,
  m,
  kappa,
```

```
  nu,
  sigma,
  L = 1,
  N = 10,
  boundary = c("neumann", "dirichlet", "periodic")
)
```

Arguments

h, m	Vectors of arguments of the covariance function.
kappa	Range parameter.
nu	Shape parameter.
sigma	Standard deviation.
L	The upper bound of the interval $[0, L]$. By default, L=1.
N	The truncation parameter.
boundary	The boundary condition. The possible conditions are "neumann" (default), "dirichlet" or "periodic".

Details

folded.matern.covariance.1d evaluates the 1d folded Matern covariance function over an interval $[0, L]$ under different boundary conditions. For periodic boundary conditions

$$C_{\mathcal{P}}(h, m) = \sum_{k=-\infty}^{\infty} (C(h - m + 2kL),$$

for Neumann boundary conditions

$$C_{\mathcal{N}}(h, m) = \sum_{k=-\infty}^{\infty} (C(h - m + 2kL) + C(h + m + 2kL)),$$

and for Dirichlet boundary conditions:

$$C_{\mathcal{D}}(h, m) = \sum_{k=-\infty}^{\infty} (C(h - m + 2kL) - C(h + m + 2kL)),$$

where $C(\cdot)$ is the Matern covariance function:

$$C(h) = \frac{\sigma^2}{2^{\nu-1}\Gamma(\nu)}(\kappa h)^{\nu} K_{\nu}(\kappa h).$$

We consider the truncation:

$$C_{\mathcal{P},N}(h, m) = \sum_{k=-N}^{N} C(h-m+2kL), C_{\mathcal{N},N}(h, m) = \sum_{k=-\infty}^{\infty} (C(h-m+2kL)+C(h+m+2kL)),$$

and

$$C_{\mathcal{D},N}(h, m) = \sum_{k=-N}^{N} (C(h - m + 2kL) - C(h + m + 2kL)).$$

Value

A matrix with the corresponding covariance values.

Examples

```
x <- seq(from = 0, to = 1, length.out = 101)
plot(x, folded.matern.covariance.1d(rep(0.5, length(x)), x,
kappa = 10, nu = 1 / 5, sigma = 1),
  type = "l", ylab = "C(h)", xlab = "h"
)
```

`folded.matern.covariance.2d`

The 2d folded Matern covariance function

Description

`folded.matern.covariance.2d` evaluates the 2d folded Matern covariance function over an interval $[0, L] \times [0, L]$.

Usage

```
folded.matern.covariance.2d(
  h,
  m,
  kappa,
  nu,
  sigma,
  L = 1,
  N = 10,
  boundary = c("neumann", "dirichlet", "periodic", "R2")
)
```

Arguments

`h, m`	Vectors with two coordinates.
`kappa`	Range parameter.
`nu`	Shape parameter.
`sigma`	Standard deviation.
`L`	The upper bound of the square $[0, L] \times [0, L]$. By default, `L=1`.
`N`	The truncation parameter.
`boundary`	The boundary condition. The possible conditions are `"neumann"` (default), `"dirichlet"`, `"periodic"` or `"R2"`.

Details

`folded.matern.covariance.2d` evaluates the 1d folded Matern covariance function over an interval $[0, L] \times [0, L]$ under different boundary conditions. For periodic boundary conditions

$$C_{\mathcal{P}}((h_1, h_2), (m_1, m_2)) = \sum_{k_2=-\infty}^{\infty} \sum_{k_1=-\infty}^{\infty} (C(\|(h_1 - m_1 + 2k_1L, h_2 - m_2 + 2k_2L)\|),$$

for Neumann boundary conditions

$$C_{\mathcal{N}}((h_1, h_2), (m_1, m_2)) = \sum_{k_2=-\infty}^{\infty} \sum_{k_1=-\infty}^{\infty} (C(\|(h_1-m_1+2k_1L, h_2-m_2+2k_2L)\|)+C(\|(h_1-m_1+2k_1L, h_2+m_2+2k_2L)\|$$

and for Dirichlet boundary conditions:

$$C_{\mathcal{D}}((h_1, h_2), (m_1, m_2)) = \sum_{k_2=-\infty}^{\infty} \sum_{k_1=-\infty}^{\infty} (C(\|(h_1-m_1+2k_1L, h_2-m_2+2k_2L)\|)-C(\|(h_1-m_1+2k_1L, h_2+m_2+2k_2L)\|$$

where $C(\cdot)$ is the Matern covariance function:

$$C(h) = \frac{\sigma^2}{2^{\nu-1}\Gamma(\nu)}(\kappa h)^{\nu} K_{\nu}(\kappa h).$$

We consider the truncation for k_1, k_2 from $-N$ to N.

Value

The correspoding covariance.

Examples

```
h <- c(0.5, 0.5)
m <- c(0.5, 0.5)
folded.matern.covariance.2d(h, m, kappa = 10, nu = 1 / 5, sigma = 1)
```

`fractional.operators` *Rational approximations of fractional operators*

Description

`fractional.operators` is used for computing an approximation, which can be used for inference and simulation, of the fractional SPDE

$$L^{\beta}(\tau u(s)) = W.$$

Here L is a differential operator, $\beta > 0$ is the fractional power, τ is a positive scalar or vector that scales the variance of the solution u, and W is white noise.

Usage

```
fractional.operators(L, beta, C, scale.factor, m = 1, tau = 1)
```

Arguments

L	A finite element discretization of the operator L.
beta	The positive fractional power.
C	The mass matrix of the finite element discretization.
scale.factor	A constant c is a lower bound for the the smallest eigenvalue of the non-discretized operator L.
m	The order of the rational approximation, which needs to be a positive integer. The default value is 1. Higer values gives a more accurate approximation, which are more computationally expensive to use for inference. Currently, the largest value of m that is implemented is 4.
tau	The constant or vector that scales the variance of the solution. The default value is 1.

Details

The approximation is based on a rational approximation of the fractional operator, resulting in an approximate model on the form

$$P_l u(s) = P_r W,$$

where $P_j = p_j(L)$ are non-fractional operators defined in terms of polynomials p_j for $j = l, r$. The order of p_r is given by m and the order of p_l is $m + m_\beta$ where m_β is the integer part of β if $\beta > 1$ and $m_\beta = 1$ otherwise.

The discrete approximation can be written as $u = P_r x$ where $x \sim N(0, Q^{-1})$ and $Q = P_l^T C^{-1} P_l$. Note that the matrices P_r and Q may be be ill-conditioned for $m > 1$. In this case, the methods in `operator.operations()` should be used for operations involving the matrices, since these methods are more numerically stable.

Value

`fractional.operators` returns an object of class "rSPDEobj". This object contains the following quantities:

Pl	The operator P_l.
Pr	The operator P_r.
C	The mass lumped mass matrix.
Ci	The inverse of C.
m	The order of the rational approximation.
beta	The fractional power.
type	String indicating the type of approximation.
Q	The matrix `t(Pl) %*% solve(C,Pl)`.
type	String indicating the type of approximation.
Pl.factors	List with elements that can be used to assemble P_l.
Pr.factors	List with elements that can be used to assemble P_r.

See Also

matern.operators(), spde.matern.operators(), matern.operators()

Examples

```
# Compute rational approximation of a Gaussian process with a
# Matern covariance function on R
kappa <- 10
sigma <- 1
nu <- 0.8

# create mass and stiffness matrices for a FEM discretization
x <- seq(from = 0, to = 1, length.out = 101)
fem <- rSPDE.fem1d(x)

# compute rational approximation of covariance function at 0.5
tau <- sqrt(gamma(nu) / (sigma^2 * kappa^(2 * nu) *
  (4 * pi)^(1 / 2) * gamma(nu + 1 / 2)))
op <- fractional.operators(
  L = fem$G + kappa^2 * fem$C, beta = (nu + 1 / 2) / 2,
  C = fem$C, scale.factor = kappa^2, tau = tau
)

v <- t(rSPDE.A1d(x, 0.5))
c.approx <- Sigma.mult(op, v)

# plot the result and compare with the true Matern covariance
plot(x, matern.covariance(abs(x - 0.5), kappa, nu, sigma),
  type = "l", ylab = "C(h)",
  xlab = "h", main = "Matern covariance and rational approximations"
)
lines(x, c.approx, col = 2)
```

---

get.inital.values.rSPDE

Initial values for log-likelihood optimization in rSPDE models with a latent stationary Gaussian Matern model

---

Description

Auxiliar function to obtain domain-based initial values for log-likelihood optimization in rSPDE models with a latent stationary Gaussian Matern model

Usage

```
get.inital.values.rSPDE(
  mesh = NULL,
  mesh.range = NULL,
```

```
  dim = NULL,
  include.nu = TRUE,
  log.scale = TRUE,
  include.tau = FALSE,
  nu_upper_bound = NULL
)
```

Arguments

mesh	An in INLA mesh
mesh.range	The range of the mesh.
dim	The dimension of the domain.
include.nu	Should we also provide an initial guess for nu?
log.scale	Should the results be provided in log scale?
include.tau	Should tau be returned instead of sigma?
nu_upper_bound	Should an upper bound for nu be considered?

Value

A vector of the form (theta_1,theta_2,theta_3) or where theta_1 is the initial guess for tau, theta_2 is the initial guess for kappa and theta_3 is the initial guess for nu.

---

get.sparsity.graph.rspde

Sparsity graph for rSPDE models

Description

Creates the sparsity graph for rSPDE models

Usage

```
get.sparsity.graph.rspde(
  mesh = NULL,
  fem_mesh_matrices = NULL,
  nu,
  force_non_integer = FALSE,
  rspde_order = 2,
  sharp = TRUE,
  dim = NULL
)
```

Arguments

mesh	An INLA mesh, optional
fem_mesh_matrices	A list containing the FEM-related matrices. The list should contain elements C, G, G_2, G_3, etc. Optional, should be provided if mesh is not provided.
nu	The smoothness parameter
force_non_integer	Should nu be treated as non_integer?
rspde_order	The order of the covariance-based rational SPDE approach.
sharp	The graph should have the correct sparsity (costs more to perform a sparsity analysis) or an upper bound for the sparsity?
dim	The dimension, optional. Should be provided if mesh is not provided.

Value

The sparsity graph for rSPDE models to be used in R-INLA interface.

---

matern.covariance *The Matern covariance function*

Description

matern.covariance evaluates the Matern covariance function

$$C(h) = \frac{\sigma^2}{2^{\nu-1}\Gamma(\nu)}(\kappa h)^\nu K_\nu(\kappa h).$$

Usage

```
matern.covariance(h, kappa, nu, sigma)
```

Arguments

h	Distances to evaluate the covariance function at.
kappa	Range parameter.
nu	Shape parameter.
sigma	Standard deviation.

Value

A vector with the values C(h).

Examples

```
x <- seq(from = 0, to = 1, length.out = 101)
plot(x, matern.covariance(abs(x - 0.5), kappa = 10, nu = 1 / 5, sigma = 1),
  type = "l", ylab = "C(h)", xlab = "h"
)
```

`matern.loglike` *Parameter-based log-likelihood for a latent Gaussian Matern model using a rational SPDE approximation*

Description

This function evaluates the log-likelihood function for a Gaussian process with a Matern covariance function, that is observed under Gaussian measurement noise: $Y_i = u(s_i) + \epsilon_i$, where ϵ_i are iid mean-zero Gaussian variables. The latent model is approximated using a rational approximation of the fractional SPDE model corresponding to the Gaussian process.

Usage

```
matern.loglike(
  kappa,
  sigma,
  nu,
  sigma.e,
  Y,
  G,
  C,
  A,
  mu = 0,
  d = 2,
  m = 1,
  type = c("covariance", "operator"),
  pivot = TRUE
)
```

Arguments

`kappa`	Range parameter of the latent process.
`sigma`	Standard deviation of the latent process.
`nu`	Shape parameter of the latent process.
`sigma.e`	The standard deviation of the measurement noise.
`Y`	The observations, either a vector or a matrix where the columns correspond to independent replicates of observations.
`G`	The stiffness matrix of a finite element discretization of the domain.

C	The mass matrix of a finite element discretization of the domain.
A	A matrix linking the measurement locations to the basis of the FEM approximation of the latent model.
mu	Expectation vector of the latent field (default = 0).
d	The dimension of the domain. The default value is 2.
m	The order of the rational approximation, which needs to be a positive integer. The default value is 1.
type	The type of the rational approximation. The options are "covariance" and "operator". The default is "covariance".
pivot	Should pivoting be used for the Cholesky decompositions? Default is TRUE

Value

The log-likelihood value.

See Also

spde.matern.loglike(), rSPDE.loglike(), matern.operators().

Examples

```
# this example illustrates how the function can be used for maximum
# likelihood estimation

set.seed(123)
# Sample a Gaussian Matern process on R using the covariance-based
# rational approximation
nu <- 0.8
kappa <- 5
sigma <- 1
sigma.e <- 0.1
n.rep <- 10
n.obs <- 100
n.x <- 51

# create mass and stiffness matrices for a FEM discretization
x <- seq(from = 0, to = 1, length.out = n.x)
fem <- rSPDE.fem1d(x)

tau <- sqrt(gamma(nu) / (sigma^2 * kappa^(2 * nu) * (4 * pi)^(1 / 2) *
gamma(nu + 1 / 2)))

# Compute the covariance-based rational approximation
op_cov <- matern.operators(
  C = fem$C, G = fem$G, nu = nu,
  kappa = kappa, sigma = sigma, d = 1, m = 2
)

# Sample the model
u <- simulate(op_cov, n.rep)
```

```
# Create some data
obs.loc <- runif(n = n.obs, min = 0, max = 1)
A <- rSPDE.A1d(x, obs.loc)
noise <- rnorm(n.obs * n.rep)
dim(noise) <- c(n.obs, n.rep)
Y <- as.matrix(A %*% u + sigma.e * noise)

# Define the negative likelihood function for optimization
# using CBrSPDE.matern.loglike
# Notice that we are also using sigma instead of tau, so it can be compared
# to matern.loglike()
mlik_cov2 <- function(theta, Y, A, C, G) {
  kappa <- exp(theta[1])
  sigma <- exp(theta[2])
  nu <- exp(theta[3])
  return(-matern.loglike(
    kappa = kappa, sigma = sigma,
    nu = nu, sigma.e = exp(theta[4]), Y = Y, A = A,
    C = fem$C, G = fem$G, d = 1
  ))
}

# The parameters can now be estimated by minimizing mlik with optim

# Choose some reasonable starting values depending on the size of the domain
theta0 <- log(c(sqrt(8), sqrt(var(c(Y))), 0.9, 0.01))

# run estimation and display the results
theta <- optim(theta0, mlik_cov2,
  Y = Y, A = A, C = C, G = G,
  method = "L-BFGS-B"
)

print(data.frame(
  kappa = c(kappa, exp(theta$par[1])), sigma = c(sigma, exp(theta$par[2])),
  nu = c(nu, exp(theta$par[3])), sigma.e = c(sigma.e, exp(theta$par[4])),
  row.names = c("Truth", "Estimates")
))


# this example illustrates how the function can be used for
# maximum likelihood estimation when using the operator-based
# rational approximation
set.seed(123)
# Sample a Gaussian Matern process on R using a rational approximation
nu <- 0.8
kappa <- 5
sigma <- 1
sigma.e <- 0.1
n.rep <- 10
n.obs <- 100
n.x <- 51
```

```
# create mass and stiffness matrices for a FEM discretization
x <- seq(from = 0, to = 1, length.out = n.x)
fem <- rSPDE.fem1d(x)

# compute rational approximation
op <- matern.operators(
  kappa = kappa, sigma = sigma, nu = nu,
  G = fem$G, C = fem$C, d = 1,
  type = "operator"
)

# Sample the model
u <- simulate(op, n.rep)

# Create some data
obs.loc <- runif(n = n.obs, min = 0, max = 1)
A <- rSPDE.A1d(x, obs.loc)
noise <- rnorm(n.obs * n.rep)
dim(noise) <- c(n.obs, n.rep)
Y <- as.matrix(A %*% u + sigma.e * noise)

# define negative likelihood function for optimization using matern.loglike
mlik <- function(theta, Y, G, C, A) {
  return(-matern.loglike(exp(theta[1]), exp(theta[2]),
    exp(theta[3]), exp(theta[4]),
    Y = Y, G = G, C = C, A = A, d = 1,
    type = "operator"
  ))
}

# The parameters can now be estimated by minimizing mlik with optim

# Choose some reasonable starting values depending on the size of the domain
theta0 <- log(c(sqrt(8), sqrt(var(c(Y))), 0.9, 0.01))

# run estimation and display the results
theta <- optim(theta0, mlik,
  Y = Y, G = fem$G, C = fem$C, A = A,
  method = "L-BFGS-B"
)

print(data.frame(
  kappa = c(kappa, exp(theta$par[1])), sigma = c(sigma, exp(theta$par[2])),
  nu = c(nu, exp(theta$par[3])), sigma.e = c(sigma.e, exp(theta$par[4])),
  row.names = c("Truth", "Estimates")
))
```

`matern.operators` *Rational approximations of stationary Gaussian Matern random fields*

Description

`matern.operators` is used for computing a rational SPDE approximation of a stationary Gaussian random fields on R^d with a Matern covariance function

$$C(h) = \frac{\sigma^2}{2^{\nu-1}\Gamma(\nu)}(\kappa h)^\nu K_\nu(\kappa h)$$

Usage

```
matern.operators(
  kappa,
  sigma,
  nu,
  G = NULL,
  C = NULL,
  d = NULL,
  mesh = NULL,
  m = 1,
  type = c("covariance", "operator"),
  compute_higher_order = FALSE,
  return_block_list = FALSE,
  type_rational_approximation = c("chebfun", "brasil", "chebfunLB")
)
```

Arguments

`kappa`	Range parameter of the covariance function.
`sigma`	Standard deviation of the covariance function.
`nu`	Shape parameter of the covariance function.
`G`	The stiffness matrix of a finite element discretization of the domain of interest. Does not need to be given if `mesh` is used.
`C`	The mass matrix of a finite element discretization of the domain of interest. Does not need to be given if `mesh` is used.
`d`	The dimension of the domain. Does not need to be given if `mesh` is used.
`mesh`	An optional inla mesh. `d`, `C` and `G` must be given if `mesh` is not given.
`m`	The order of the rational approximation, which needs to be a positive integer. The default value is 1.
`type`	The type of the rational approximation. The options are "covariance" and "operator". The default is "covariance".
`compute_higher_order`	Logical. Should the higher order finite element matrices be computed?
`return_block_list`	Logical. For `type = "covariance"`, should the block parts of the precision matrix be returned separately as a list?
`type_rational_approximation`	Which type of rational approximation should be used? The current types are "chebfun", "brasil" or "chebfunLB".

Details

If type is "covariance", we use the covariance-based rational approximation of the fractional operator. In the SPDE approach, we model u as the solution of the following SPDE:

$$L^{\alpha/2}(\tau u) = \mathcal{W},$$

where $L = -\Delta + \kappa^2 I$ and $\mathcal{W}$ is the standard Gaussian white noise. The covariance operator of u is given by $L^{-\alpha}$. Now, let L_h be a finite-element approximation of L. We can use a rational approximation of order m on $L_h^{-\alpha}$ to obtain the following approximation:

$$L_{h,m}^{-\alpha} = L_h^{-m_\alpha} p(L_h^{-1}) q(L_h^{-1})^{-1},$$

where $m_\alpha = \lfloor \alpha \rfloor$, p and q are polynomials arising from such rational approximation. From this approximation we construct an approximate precision matrix for u.

If type is "operator", the approximation is based on a rational approximation of the fractional operator $(\kappa^2 - \Delta)^\beta$, where $\beta = (\nu + d/2)/2$. This results in an approximate model of the form

$$P_l u(s) = P_r W,$$

where $P_j = p_j(L)$ are non-fractional operators defined in terms of polynomials p_j for $j = l, r$. The order of p_r is given by m and the order of p_l is $m + m_\beta$ where m_β is the integer part of β if $\beta > 1$ and $m_\beta = 1$ otherwise.

The discrete approximation can be written as $u = P_r x$ where $x \sim N(0, Q^{-1})$ and $Q = P_l^T C^{-1} P_l$. Note that the matrices P_r and Q may be be ill-conditioned for $m > 1$. In this case, the methods in operator.operations() should be used for operations involving the matrices, since these methods are more numerically stable.

Value

If type is "covariance", then matern.operators returns an object of class "CBrSPDEobj". This object is a list containing the following quantities:

C	The mass lumped mass matrix.
Ci	The inverse of C.
GCi	The stiffness matrix G times Ci
Gk	The stiffness matrix G along with the higher-order FEM-related matrices G2, G3, etc.
fem_mesh_matrices	A list containing the mass lumped mass matrix, the stiffness matrix and the higher-order FEM-related matrices.
m	The order of the rational approximation.
alpha	The fractional power of the precision operator.
type	String indicating the type of approximation.
d	The dimension of the domain.
nu	Shape parameter of the covariance function.
kappa	Range parameter of the covariance function

tau	Scale parameter of the covariance function.
sigma	Standard deviation of the covariance function.
type	String indicating the type of approximation.

If type is "operator", then matern.operators returns an object of class "rSPDEobj". This object contains the quantities listed in the output of fractional.operators(), the G matrix, the dimension of the domain, as well as the parameters of the covariance function.

See Also

fractional.operators(), spde.matern.operators(), matern.operators()

Examples

```
# Compute the covariance-based rational approximation of a
# Gaussian process with a Matern covariance function on R
kappa <- 10
sigma <- 1
nu <- 0.8

# create mass and stiffness matrices for a FEM discretization
nobs <- 101
x <- seq(from = 0, to = 1, length.out = 101)
fem <- rSPDE.fem1d(x)

# compute rational approximation of covariance function at 0.5
op_cov <- matern.operators(
  C = fem$C, G = fem$G, nu = nu,
  kappa = kappa, sigma = sigma, d = 1, m = 2
)

v <- t(rSPDE.A1d(x, 0.5))
# Compute the precision matrix
Q <- op_cov$Q
# A matrix here is the identity matrix
A <- Diagonal(nobs)
# We need to concatenate 3 A's since we are doing a covariance-based rational
# approximation of order 2
Abar <- cbind(A, A, A)
w <- rbind(v, v, v)
# The approximate covariance function:
c_cov.approx <- (Abar) %*% solve(Q, w)
c.true <- folded.matern.covariance.1d(rep(0.5, length(x)),
   abs(x), kappa, nu, sigma)

# plot the result and compare with the true Matern covariance
plot(x, c.true,
  type = "l", ylab = "C(h)",
  xlab = "h", main = "Matern covariance and rational approximations"
)
lines(x, c_cov.approx, col = 2)
```

```
# Compute the operator-based rational approximation of a Gaussian
# process with a Matern covariance function on R
kappa <- 10
sigma <- 1
nu <- 0.8

# create mass and stiffness matrices for a FEM discretization
x <- seq(from = 0, to = 1, length.out = 101)
fem <- rSPDE.fem1d(x)

# compute rational approximation of covariance function at 0.5
op <- matern.operators(
  kappa = kappa, sigma = sigma, nu = nu,
  G = fem$G, C = fem$C, d = 1,
  type = "operator"
)

v <- t(rSPDE.A1d(x, 0.5))
c.approx <- Sigma.mult(op, v)
c.true <- folded.matern.covariance.1d(rep(0.5, length(x)),
  abs(x), kappa, nu, sigma)

# plot the result and compare with the true Matern covariance
plot(x, c.true,
  type = "l", ylab = "C(h)",
  xlab = "h", main = "Matern covariance and rational approximation"
)
lines(x, c.approx, col = 2)
```

`operator.operations` *Operations with the Pr and Pl operators*

Description

Functions for multiplying and solving with the P_r and P_l operators as well as the latent precision matrix $Q = P_l C^{-1} P_l$ and covariance matrix $\Sigma = P_r Q^{-1} P_r^T$. These operations are done without first assembling P_r, P_l in order to avoid numerical problems caused by ill-conditioned matrices.

Usage

```
Pr.mult(obj, v, transpose = FALSE)

Pr.solve(obj, v, transpose = FALSE)

Pl.mult(obj, v, transpose = FALSE)

Pl.solve(obj, v, transpose = FALSE)
```

```
Q.mult(obj, v)

Q.solve(obj, v)

Qsqrt.mult(obj, v, transpose = FALSE)

Qsqrt.solve(obj, v, transpose = FALSE)

Sigma.mult(obj, v)

Sigma.solve(obj, v)
```

Arguments

obj	rSPDE object
v	vector to apply the operation to
transpose	set to TRUE if the operation should be performed with the transposed object

Details

`Pl.mult`, `Pr.mult`, and `Q.mult` multiplies the vector with the respective object. Changing `mult` to `solve` in the function names multiplies the vector with the inverse of the object. `Qsqrt.mult` and `Qsqrt.solve` performs the operations with the square-root type object $Q_r = C^{-1/2}P_l$ defined so that $Q = Q_r^T Q_r$.

Value

A vector with the values of the operation

---

plot.rspde.result *Posterior plots for field parameters for an* `inla_rspde` *model from a* `rspde.result` *object*

Description

Posterior plots for rSPDE field parameters in their original scales.

Usage

```
## S3 method for class 'rspde.result'
plot(
  x,
  which = c("tau", "kappa", "nu"),
  caption = list("Posterior density for tau", "Posterior density for kappa",
    "Posterior density for nu"),
  sub.caption = NULL,
  type_plot = "l",
```

```
 ask = prod(graphics::par("mfcol")) < length(which) && grDevices::dev.interactive(),
  main = "",
  cex.oma.main = 1.25,
  cex.caption = 1,
  ylab = "Density",
  xlab = "x",
  ...
)
```

Arguments

x	A rspde.result object.
which	For which parameters the posterior should be plotted?
caption	captions to appear above the plots; character vector or list of valid graphics annotations. Can be set to "" or NA to suppress all captions.
sub.caption	common title-above the figures if there are more than one.
type_plot	what type of plot should be drawn. The default is 'l'.
ask	logical; if TRUE, the user is asked before each plot.
main	character; title to be placed at each plot additionally (and above) all captions.
cex.oma.main	controls the size of the sub.caption only if that is above the figures when there is more than one.
cex.caption	controls the size of caption.
ylab	Label for y axis.
xlab	Label for x axis.
...	Additional arguments.

Value

Called for its side effects.

Examples

```
 #tryCatch version
tryCatch({
if (requireNamespace("INLA", quietly = TRUE)){
library(INLA)

set.seed(123)

m <- 100
loc_2d_mesh <- matrix(runif(m * 2), m, 2)
mesh_2d <- inla.mesh.2d(
  loc = loc_2d_mesh,
  cutoff = 0.05,
  max.edge = c(0.1, 0.5)
)
sigma <- 0.01
```

```
range <- 0.2
nu <- 0.8
kappa <- sqrt(8 * nu) / range
op <- matern.operators(
  mesh = mesh_2d, nu = nu,
  kappa = kappa, sigma = sigma, m = 2
)
u <- simulate(op)
A <- inla.spde.make.A(
  mesh = mesh_2d,
  loc = loc_2d_mesh
)
sigma.e <- 0.1
y <- A %*% u + rnorm(m) * sigma.e
Abar <- rspde.make.A(mesh = mesh_2d, loc = loc_2d_mesh)
mesh.index <- rspde.make.index(name = "field", mesh = mesh_2d)
st.dat <- inla.stack(
  data = list(y = as.vector(y)),
  A = Abar,
  effects = mesh.index
)
rspde_model <- rspde.matern(
  mesh = mesh_2d,
  nu_upper_bound = 2
)
f <- y ~ -1 + f(field, model = rspde_model)
rspde_fit <- inla(f,
  data = inla.stack.data(st.dat),
  family = "gaussian",
  control.predictor =
    list(A = inla.stack.A(st.dat)),
           inla.mode = "experimental"
)
result <- rspde.result(rspde_fit, "field", rspde_model)
plot(result)
}
#stable.tryCatch
}, error = function(e){print("Could not run the example")})
```

`precision` *Get the precision matrix of CBrSPDEobj objects*

Description

Function to get the precision matrix of a CBrSPDEobj object

Usage

```
precision(object, ...)
```

```
## S3 method for class 'CBrSPDEobj'
precision(
  object,
  user_nu = NULL,
  user_kappa = NULL,
  user_sigma = NULL,
  user_m = NULL,
  ...
)
```

Arguments

object	The covariance-based rational SPDE approximation, computed using matern.operators()
...	Currently not used.
user_nu	If non-null, update the shape parameter of the covariance function.
user_kappa	If non-null, update the range parameter of the covariance function.
user_sigma	If non-null, update the standard deviation of the covariance function.
user_m	If non-null, update the order of the rational approximation, which needs to be a positive integer.

Value

The precision matrix.

See Also

simulate.CBrSPDEobj(), matern.operators()

Examples

```
# Compute the covariance-based rational approximation of a
# Gaussian process with a Matern covariance function on R
kappa <- 10
sigma <- 1
nu <- 0.8

# create mass and stiffness matrices for a FEM discretization
x <- seq(from = 0, to = 1, length.out = 101)
fem <- rSPDE.fem1d(x)

# compute rational approximation of covariance function at 0.5
tau <- sqrt(gamma(nu) / (sigma^2 * kappa^(2 * nu) *
(4 * pi)^(1 / 2) * gamma(nu + 1 / 2)))
op_cov <- matern.operators(
  C = fem$C, G = fem$G, nu = nu,
  kappa = kappa, sigma = sigma, d = 1, m = 2
)
```

```
# Get the precision matrix:
prec_matrix <- precision(op_cov)
```

`predict.CBrSPDEobj` *Prediction of a fractional SPDE using the covariance-based rational SPDE approximation*

Description

The function is used for computing kriging predictions based on data $Y_i = u(s_i) + \epsilon_i$, where ϵ is mean-zero Gaussian measurement noise and $u(s)$ is defined by a fractional SPDE $(\kappa^2 I - \Delta)^{\alpha/2}(\tau u(s)) = W$, where W is Gaussian white noise and $\alpha = \nu + d/2$, where d is the dimension of the domain.

Usage

```
## S3 method for class 'CBrSPDEobj'
predict(
  object,
  A,
  Aprd,
  Y,
  sigma.e,
  mu = 0,
  compute.variances = FALSE,
  pivot = TRUE,
  ...
)
```

Arguments

`object`	The covariance-based rational SPDE approximation, computed using `matern.operators()`
`A`	A matrix linking the measurement locations to the basis of the FEM approximation of the latent model.
`Aprd`	A matrix linking the prediction locations to the basis of the FEM approximation of the latent model.
`Y`	A vector with the observed data, can also be a matrix where the columns are observations of independent replicates of u.
`sigma.e`	The standard deviation of the Gaussian measurement noise. Put to zero if the model does not have measurement noise.
`mu`	Expectation vector of the latent field (default = 0).
`compute.variances`	Set to also TRUE to compute the kriging variances.
`pivot`	Should pivoting be used on the Cholesky decompositions?
`...`	further arguments passed to or from other methods.

Value

A list with elements

mean	The kriging predictor (the posterior mean of u|Y).
variance	The posterior variances (if computed).

Examples

```
set.seed(123)
# Sample a Gaussian Matern process on R using a rational approximation
kappa <- 10
sigma <- 1
nu <- 0.8
sigma.e <- 0.3

# create mass and stiffness matrices for a FEM discretization
x <- seq(from = 0, to = 1, length.out = 101)
fem <- rSPDE.fem1d(x)

tau <- sqrt(gamma(nu) / (sigma^2 * kappa^(2 * nu) *
       (4 * pi)^(1 / 2) * gamma(nu + 1 / 2)))

# Compute the covariance-based rational approximation
op_cov <- matern.operators(
  C = fem$C, G = fem$G, nu = nu,
  kappa = kappa, sigma = sigma, d = 1, m = 2
)

# Sample the model
u <- simulate(op_cov)

# Create some data
obs.loc <- runif(n = 10, min = 0, max = 1)
A <- rSPDE.A1d(x, obs.loc)
Y <- as.vector(A %*% u + sigma.e * rnorm(10))

# compute kriging predictions at the FEM grid
A.krig <- rSPDE.A1d(x, x)
u.krig <- predict(op_cov,
  A = A, Aprd = A.krig, Y = Y, sigma.e = sigma.e,
  compute.variances = TRUE
)

plot(obs.loc, Y,
  ylab = "u(x)", xlab = "x", main = "Data and prediction",
  ylim = c(
    min(u.krig$mean - 2 * sqrt(u.krig$variance)),
    max(u.krig$mean + 2 * sqrt(u.krig$variance))
  )
)
lines(x, u.krig$mean)
lines(x, u.krig$mean + 2 * sqrt(u.krig$variance), col = 2)
```

```
lines(x, u.krig$mean - 2 * sqrt(u.krig$variance), col = 2)
```

`predict.rSPDEobj` *Prediction of a fractional SPDE using a rational SPDE approximation*

Description

The function is used for computing kriging predictions based on data $Y_i = u(s_i) + \epsilon_i$, where ϵ is mean-zero Gaussian measurement noise and $u(s)$ is defined by a fractional SPDE $L^\beta u(s) = W$, where W is Gaussian white noise.

Usage

```
## S3 method for class 'rSPDEobj'
predict(object, A, Aprd, Y, sigma.e, compute.variances = FALSE, ...)
```

Arguments

`object`	The rational SPDE approximation, computed using `fractional.operators()`, `matern.operators()`, or `spde.matern.operators()`.
`A`	A matrix linking the measurement locations to the basis of the FEM approximation of the latent model.
`Aprd`	A matrix linking the prediction locations to the basis of the FEM approximation of the latent model.
`Y`	A vector with the observed data, can also be a matrix where the columns are observations of independent replicates of u.
`sigma.e`	The standard deviation of the Gaussian measurement noise. Put to zero if the model does not have measurement noise.
`compute.variances`	Set to also TRUE to compute the kriging variances.
`...`	further arguments passed to or from other methods.

Value

A list with elements

`mean`	The kriging predictor (the posterior mean of u|Y).
`variance`	The posterior variances (if computed).

Examples

```
# Sample a Gaussian Matern process on R using a rational approximation
kappa <- 10
sigma <- 1
nu <- 0.8
sigma.e <- 0.3

# create mass and stiffness matrices for a FEM discretization
x <- seq(from = 0, to = 1, length.out = 101)
fem <- rSPDE.fem1d(x)

# compute rational approximation
op <- matern.operators(
  kappa = kappa, sigma = sigma,
  nu = nu, G = fem$G, C = fem$C, d = 1
)

# Sample the model
u <- simulate(op)

# Create some data
obs.loc <- runif(n = 10, min = 0, max = 1)
A <- rSPDE.A1d(x, obs.loc)
Y <- as.vector(A %*% u + sigma.e * rnorm(10))

# compute kriging predictions at the FEM grid
A.krig <- rSPDE.A1d(x, x)
u.krig <- predict(op,
  A = A, Aprd = A.krig, Y = Y, sigma.e = sigma.e,
  compute.variances = TRUE
)

plot(obs.loc, Y,
  ylab = "u(x)", xlab = "x", main = "Data and prediction",
  ylim = c(
    min(u.krig$mean - 2 * sqrt(u.krig$variance)),
    max(u.krig$mean + 2 * sqrt(u.krig$variance))
  )
)
lines(x, u.krig$mean)
lines(x, u.krig$mean + 2 * sqrt(u.krig$variance), col = 2)
lines(x, u.krig$mean - 2 * sqrt(u.krig$variance), col = 2)
```

`rational.order` *Get the order of rational approximation.*

Description

Get the order of rational approximation.

Usage

```
rational.order(object)
```

Arguments

object	A CBrSPDEobj object or an inla_rspde object.

Value

The order of rational approximation.

---

rational.order<- *Changing the order of the rational approximation*

Description

Changing the order of the rational approximation

Usage

```
rational.order(x) <- value
```

Arguments

x	A CBrSPDE or an rpsde.inla object
value	The order of rational approximation.

Value

An object of the same class with the new order of rational approximation.

---

rational.type *Get type of rational approximation.*

Description

Get type of rational approximation.

Usage

```
rational.type(object)
```

Arguments

object	A CBrSPDEobj object or an inla_rspde object.

Value

The type of rational approximation.

`rational.type<-` *Changing the type of the rational approximation*

Description

Changing the type of the rational approximation

Usage

```
rational.type(x) <- value
```

Arguments

`x`	A `CBrSPDE` or an `rpsde.inla` object
`value`	The type of rational approximation. The current options are "chebfun", "brasil" and "chebfunLB"

Value

An object of the same class with the new rational approximation.

`require.nowarnings` *Warnings free loading of add-on packages*

Description

Turn off all warnings for require(), to allow clean completion of examples that require unavailable Suggested packages.

Usage

```
require.nowarnings(package, lib.loc = NULL, character.only = FALSE)
```

Arguments

`package`	The name of a package, given as a character string.
`lib.loc`	a character vector describing the location of R library trees to search through, or `NULL`. The default value of `NULL` corresponds to all libraries currently known to `.libPaths()`. Non-existent library trees are silently ignored.
`character.only`	a logical indicating whether `package` can be assumed to be a character string.

Details

`require(package)` acts the same as `require(package, quietly = TRUE)` but with warnings turned off. In particular, no warning or error is given if the package is unavailable. Most cases should use `requireNamespace(package, quietly = TRUE)` instead, which doesn't produce warnings.

Value

`require.nowarnings` returns (invisibly) `TRUE` if it succeeds, otherwise `FALSE`

See Also

`require()`

Examples

```
## This should produce no output:
if (require.nowarnings(nonexistent)) {
  message("Package loaded successfully")
}
```

---

rSPDE *Rational approximations of fractional SPDEs.*

Description

`rSPDE` is used for approximating fractional elliptic SPDEs

$$L^{\beta}(\tau u(s)) = W,$$

where L is a differential operator and $\beta > 0$ is a general fractional power.

Details

The approximation is based on a rational approximation of the fractional operator, and allows for computationally efficient inference and simulation.

The main functions for computing rational approximation objects are:

- `fractional.operators()`works for general rational operators
- `matern.operators()` works for random fields with stationary Matern covariance functions
- `spde.matern.operators()` works for random fields with defined as solutions to a possibly non-stationary Matern-type SPDE model.
- `rspde.matern()` R-INLA implementation of the covariance-based rational approximation for random fields with stationary Matern covariance functions

Basic statistical operations such as likelihood evaluations (see `[rSPDE.loglike]`, `[rSPDE.matern.loglike]`) and kriging predictions (see `[predict.rSPDEobj]`, `[predict.CBrSPDEobj]`) using the rational approximations are also implemented.

For illustration purposes, the package contains a simple FEM implementation for models on R. For spatial models, the FEM implementation in the `R-INLA` package is recommended.

For a more detailed introduction to the package, see the rSPDE Vignettes.

`rSPDE.A1d` *Observation matrix for finite element discretization on R*

Description

A finite element discretization on R can be written as $u(s) = \sum_i^n u_i \varphi_i(s)$ where $\varphi_i(s)$ is a piecewise linear "hat function" centered at location x_i. This function computes an $m \times n$ matrix A that links the basis function in the expansion to specified locations $s = (s_1, \ldots, s_m)$ in the domain through $A_i j = \varphi_j(s_i)$.

Usage

```
rSPDE.A1d(x, loc)
```

Arguments

`x`	The locations of the nodes in the FEM discretization.
`loc`	The locations $(s_1, \ldots, s_m)$

Value

The sparse matrix `A`.

Author(s)

David Bolin <davidbolin@gmail.com>

See Also

`rSPDE.fem1d()`

Examples

```
# create mass and stiffness matrices for a FEM discretization on [0,1]
x <- seq(from = 0, to = 1, length.out = 101)
fem <- rSPDE.fem1d(x)

# create the observation matrix for some locations in the domain
obs.loc <- runif(n = 10, min = 0, max = 1)
A <- rSPDE.A1d(x, obs.loc)
```

`rSPDE.construct.matern.loglike` *Constructor of Matern loglikelihood functions.*

Description

This function returns a log-likelihood function for a Gaussian process with a Matern covariance function, that is observed under Gaussian measurement noise: $Y_i = u(s_i) + \epsilon_i$, where ϵ_i are iid mean-zero Gaussian variables. The latent model is approximated using the a rational approximation of the fractional SPDE model corresponding to the Gaussian process.

Usage

```
rSPDE.construct.matern.loglike(
  object,
  Y,
  A,
  sigma.e = NULL,
  mu = 0,
  user_nu = NULL,
  user_kappa = NULL,
  user_sigma = NULL,
  user_m = NULL,
  log_scale = TRUE,
  return_negative_likelihood = TRUE,
  pivot = TRUE
)
```

Arguments

`object`	The rational SPDE approximation, computed using `matern.operators()`
`Y`	The observations, either a vector or a matrix where the columns correspond to independent replicates of observations.
`A`	An observation matrix that links the measurement location to the finite element basis.
`sigma.e`	IF non-null, the standard deviation of the measurement noise will be kept fixed in the returned likelihood.
`mu`	Expectation vector of the latent field (default = 0).
`user_nu`	If non-null, the shape parameter will be kept fixed in the returned likelihood.
`user_kappa`	If non-null, the range parameter will be kept fixed in the returned likelihood.
`user_sigma`	If non-null, the standard deviation will be kept fixed in the returned likelihood.
`user_m`	If non-null, update the order of the rational approximation, which needs to be a positive integer.
`log_scale`	Should the parameters be evaluated in log-scale?

return_negative_likelihood
: Return minus the likelihood to turn the maximization into a minimization?

pivot
: Should pivoting be used for the Cholesky decompositions? Default is TRUE

Value

The log-likelihood function. The parameters of the returned function are given in the order sigma, kappa, nu, sigma.e, whenever they are available.

See Also

matern.operators(), predict.CBrSPDEobj()

Examples

```
# this example illustrates how the function can be used for maximum
# likelihood estimation

set.seed(123)
# Sample a Gaussian Matern process on R using a rational approximation
nu <- 0.8
kappa <- 5
sigma <- 1
sigma.e <- 0.1
n.rep <- 10
n.obs <- 100
n.x <- 51

# create mass and stiffness matrices for a FEM discretization
x <- seq(from = 0, to = 1, length.out = n.x)
fem <- rSPDE.fem1d(x)

tau <- sqrt(gamma(nu) / (sigma^2 * kappa^(2 * nu) *
(4 * pi)^(1 / 2) * gamma(nu + 1 / 2)))

# Compute the covariance-based rational approximation
op_cov <- matern.operators(
  C = fem$C, G = fem$G, nu = nu,
  kappa = kappa, sigma = sigma, d = 1, m = 2
)

# Sample the model
u <- simulate(op_cov, n.rep)

# Create some data
obs.loc <- runif(n = n.obs, min = 0, max = 1)
A <- rSPDE.A1d(x, obs.loc)
noise <- rnorm(n.obs * n.rep)
dim(noise) <- c(n.obs, n.rep)
Y <- as.matrix(A %*% u + sigma.e * noise)

# Define the negative likelihood function for optimization
```

```
# using CBrSPDE.matern.loglike

# Notice that we are also using sigma instead of tau, so it can be compared
# to matern.loglike()
loglike <- rSPDE.construct.matern.loglike(op_cov, Y, A)

# The parameters can now be estimated by minimizing mlik with optim

# Choose some reasonable starting values depending on the size of the domain
theta0 <- log(c(sqrt(8), 1 / sqrt(var(c(Y))), 0.9, 0.01))

# run estimation and display the results
theta <- optim(theta0, loglike,
  method = "L-BFGS-B"
)

print(data.frame(
  kappa = c(kappa, exp(theta$par[1])), sigma = c(sigma, exp(theta$par[2])),
  nu = c(nu, exp(theta$par[3])), sigma.e = c(sigma.e, exp(theta$par[4])),
  row.names = c("Truth", "Estimates")
))
```

`rSPDE.fem1d` *Finite element calculations for problems on R*

Description

This function computes mass and stiffness matrices for a FEM approximation on R, assuming Neumann boundary conditions. These matrices are needed when discretizing the operators in rational approximations.

Usage

```
rSPDE.fem1d(x)
```

Arguments

`x`	Locations of the nodes in the FEM approximation.

Value

The function returns a list with the following elements

`G`	The stiffness matrix.
`C`	The mass matrix.

Author(s)

David Bolin <davidbolin@gmail.com>

See Also

rSPDE.A1d()

Examples

```
# create mass and stiffness matrices for a FEM discretization on [0,1]
x <- seq(from = 0, to = 1, length.out = 101)
fem <- rSPDE.fem1d(x)
```

---

rSPDE.loglike *Object-based log-likelihood function for latent Gaussian fractional SPDE model*

Description

This function evaluates the log-likelihood function for a fractional SPDE model $L^\beta u(s) = W$ that is observed under Gaussian measurement noise: $Y_i = u(s_i) + \epsilon_i$, where ϵ_i are iid mean-zero Gaussian variables and $x(s) = \mu(s) + u(s)$, where $\mu(s)$ is the expectation vector of the latent field.

Usage

```
rSPDE.loglike(obj, Y, A, sigma.e, mu = 0)
```

Arguments

obj	The rational SPDE approximation, computed using fractional.operators(), matern.operators(), or spde.matern.operators().
Y	The observations, either a vector or a matrix where the columns correspond to independent replicates of observations.
A	An observation matrix that links the measurement location to the finite element basis.
sigma.e	The standard deviation of the measurement noise.
mu	Expectation vector of the latent field (default = 0).

Value

The log-likelihood value.

Note

This example below shows how the function can be used to evaluate the likelihood of a latent Matern model. Se matern.loglike() for an example of how this can be used for maximum likelihood estimation.

See Also

matern.loglike(), spde.matern.loglike()

Examples

```
# Sample a Gaussian Matern process on R using a rational approximation
kappa <- 10
sigma <- 1
nu <- 0.8
sigma.e <- 0.3

# create mass and stiffness matrices for a FEM discretization
x <- seq(from = 0, to = 1, length.out = 101)
fem <- rSPDE.fem1d(x)

# compute rational approximation
op <- matern.operators(
  kappa = kappa, sigma = sigma, nu = nu,
  G = fem$G, C = fem$C, d = 1,
  type = "operator"
)

# Sample the model
u <- simulate(op)

# Create some data
obs.loc <- runif(n = 10, min = 0, max = 1)
A <- rSPDE.A1d(x, obs.loc)
Y <- as.vector(A %*% u + sigma.e * rnorm(10))

# compute log-likelihood of the data
lik1 <- rSPDE.loglike(op, Y, A, sigma.e)
cat(lik1)
```

---

rspde.make.A *Observation/prediction matrices for rSPDE models.*

Description

Constructs observation/prediction weight matrices for rSPDE models based on `inla.mesh` or `inla.mesh.1d` objects.

Usage

```
rspde.make.A(
  mesh = NULL,
  loc = NULL,
  A = NULL,
  dim = NULL,
```

```
  rspde_order = 2,
  nu = NULL,
  index = NULL,
  group = NULL,
  repl = 1L,
  n.group = NULL,
  n.repl = NULL
)
```

Arguments

mesh	An inla.mesh or an inla.mesh.1d object.
loc	Locations, needed if an INLA mesh is provided
A	The A matrix from the standard SPDE approach, such as the matrix returned by inla.spde.make.A. Should only be provided if mesh is not provided.
dim	the dimension. Should only be provided if an mesh is not provided.
rspde_order	The order of the covariance-based rational SPDE approach.
nu	If NULL, then the model will assume that nu will be estimated. If nu is fixed, you should provide the value of nu.
index	For each observation/prediction value, an index into loc. Default is seq_len(nrow(A.loc)).
group	For each observation/prediction value, an index into the group model.
repl	For each observation/prediction value, the replicate index.
n.group	The size of the group model.
n.repl	The total number of replicates.

Value

The A matrix for rSPDE models.

Examples

```
 #tryCatch version
tryCatch({
if (requireNamespace("INLA", quietly = TRUE)){
library(INLA)

set.seed(123)
loc <- matrix(runif(100 * 2) * 100, 100, 2)
mesh <- inla.mesh.2d(
  loc = loc,
  cutoff = 50,
  max.edge = c(50, 500)
)
A <- rspde.make.A(mesh, loc = loc, rspde_order = 3)
}
#stable.tryCatch
}, error = function(e){print("Could not run the example")})
```

`rspde.make.index` *rSPDE model index vector generation*

Description

Generates a list of named index vectors for an rSPDE model.

Usage

```
rspde.make.index(
  name,
  n.spde = NULL,
  n.group = 1,
  n.repl = 1,
  mesh = NULL,
  rspde_order = 2,
  nu = NULL,
  dim = NULL
)
```

Arguments

`name`	A character string with the base name of the effect.
`n.spde`	The number of basis functions in the mesh model.
`n.group`	The size of the group model.
`n.repl`	The total number of replicates.
`mesh`	An `inla.mesh` or an `inla.mesh.1d` object.
`rspde_order`	The order of the rational approximation
`nu`	If `NULL`, then the model will assume that nu will be estimated. If nu is fixed, you should provide the value of nu.
`dim`	the dimension of the domain. Should only be provided if `mesh` is not provided.

Value

A list of named index vectors.

`name`	Indices into the vector of latent variables
`name.group`	'group' indices
`name.repl`	Indices for replicates

Examples

```
 #tryCatch version
tryCatch({
if (requireNamespace("INLA", quietly = TRUE)){
library(INLA)

set.seed(123)

m <- 100
loc_2d_mesh <- matrix(runif(m * 2), m, 2)
mesh_2d <- inla.mesh.2d(
  loc = loc_2d_mesh,
  cutoff = 0.05,
  max.edge = c(0.1, 0.5)
)
sigma <- 0.01
range <- 0.2
nu <- 0.8
kappa <- sqrt(8 * nu) / range
op <- matern.operators(
  mesh = mesh_2d, nu = nu,
  kappa = kappa, sigma = sigma, m = 2
)
u <- simulate(op)
A <- inla.spde.make.A(
  mesh = mesh_2d,
  loc = loc_2d_mesh
)
sigma.e <- 0.1
y <- A %*% u + rnorm(m) * sigma.e
Abar <- rspde.make.A(mesh = mesh_2d, loc = loc_2d_mesh)
mesh.index <- rspde.make.index(name = "field", mesh = mesh_2d)
st.dat <- inla.stack(
  data = list(y = as.vector(y)),
  A = Abar,
  effects = mesh.index
)
rspde_model <- rspde.matern(
  mesh = mesh_2d,
  nu_upper_bound = 2
)
f <- y ~ -1 + f(field, model = rspde_model)
rspde_fit <- inla(f,
  data = inla.stack.data(st.dat),
  family = "gaussian",
  control.predictor =
    list(A = inla.stack.A(st.dat)),
           inla.mode = "experimental"
)
result <- rspde.result(rspde_fit, "field", rspde_model)
plot(result)
}
```

```
#stable.tryCatch
}, error = function(e){print("Could not run the example")})
```

`rspde.matern` *Matern rSPDE model object for INLA*

Description

Creates an INLA object for a stationary Matern model with general smoothness parameter.

Usage

```
rspde.matern(
  mesh,
  nu_upper_bound = 4,
  rspde_order = 2,
  nu = NULL,
  sharp = TRUE,
  debug = FALSE,
  optimize = TRUE,
  prior.kappa = NULL,
  prior.nu = NULL,
  prior.tau = NULL,
  start.lkappa = NULL,
  start.nu = NULL,
  start.ltau = NULL,
  prior.nu.dist = c("beta", "lognormal"),
  nu.prec.inc = 1,
  type.rational.approx = c("chebfun", "brasil", "chebfunLB")
)
```

Arguments

`mesh`	The mesh to build the model. Should be an `inla.mesh` or an `inla.mesh.1d` object.
`nu_upper_bound`	Upper bound for the smoothness parameter.
`rspde_order`	The order of the covariance-based rational SPDE approach.
`nu`	If nu is set to a parameter, nu will be kept fixed and will not be estimated. If nu is `NULL`, it will be estimated.
`sharp`	The sparsity graph should have the correct sparsity (costs more to perform a sparsity analysis) or an upper bound for the sparsity? If `TRUE`, the graph will have the correct sparsity.
`debug`	INLA debug argument.
`optimize`	Should the model be optimized? In this case the sparsities of the matrices will be analyzed.

`prior.kappa` a `list` containing the elements `meanlog` and `sdlog`, that is, the mean and standard deviation on the log scale.

`prior.nu` a list containing the elements `mean` and `prec` for beta distribution, or `loglocation` and `logscale` for a truncated lognormal distribution. `loglocation` stands for the location parameter of the truncated lognormal distribution in the log scale. `prec` stands for the precision of a beta distribution. `logscale` stands for the scale of the truncated lognormal distribution on the log scale. Check details below.

`prior.tau` a list containing the elements `meanlog` and `sdlog`, that is, the mean and standard deviation on the log scale.

`start.lkappa` Starting value for log of kappa.

`start.nu` Starting value for nu.

`start.ltau` Starting value for log of tau.

`prior.nu.dist` The distribution of the smoothness parameter. The current options are "beta" or "lognormal". The default is "beta".

`nu.prec.inc` Amount to increase the precision in the beta prior distribution. Check details below.

`type.rational.approx`
Which type of rational approximation should be used? The current types are "chebfun", "brasil" or "chebfunLB".

Details

This function constructs a stationary Matern rSPDE model to be used with the INLA interface. The parameters are the range parameter κ, the smoothness parameter ν and the variance rescaling parameter τ.

For this model, an upper bound for the smoothness parameter ν should be given. It is given by the `nu_upper_bound` argument.

It is very important to notice that the larger the value of `nu_upper_bound` the higher the computational cost to fit the model. So, it is generally best to initially fit a model with a small value of `nu_upper_bound` and increase it only if it is really needed (for instance, if the estimated smoothness parameter was very close to `nu_upper_bound`).

The following parameterization is used:

$$\log(\tau) = \theta_1,$$

$$\log(\kappa) = \theta_2$$

and for θ_3 we can have a beta prior or a truncated lognormal prior distribution. In each case, the prior distribution has support on the interval $(0, \nu_{UB})$, where ν_{UB} is `nu_upper_bound`. Then, the following parameterization is considered:

$$\log\left(\frac{\nu}{\nu_{UB} - \nu}\right) = \theta_3.$$

By default, an optimized version of this model is considered. The optimized version is generally much faster for larger datasets, however it takes more time to build the model as the sparsity of the

graph should be analyzed. However, for small datasets, it is possible that the time taken to analyze sparsity plus fitting the model is larger than the time taken to fit an unoptimized model. So, for a small dataset it might be convenient to set `optimize=FALSE`.

A way to use the optimized version but reduce the cost of sparsity analysis is to set `sharp` to `FALSE`. However, it should increase the cost of fitting the model. Therefore, one usually would not benefit from setting the `sharp` argument to `FALSE` when fitting the model to large datasets.

Finally, when considering a beta prior, the beta distribution will be parameterized in terms of its mean, say μ and a precision parameter ϕ, which is such that the variance of the beta distribution is given by $\mu(\nu_{UB} - \mu)/(1 + \phi)$. The mean of the beta prior is determined by the `prior.nu$mean`, whereas the precision parameter is determined by the `prior.nu$prec`. If `prior.nu$prec` is `NULL` (which is the default case), the precision parameter is taken as

$$\phi = \max\left\{\frac{\nu_{UB}}{\mu}, \frac{\nu_{UB}}{\nu_{UB} - \mu}\right\} + \text{nu.prec.inc},$$

where μ is the prior mean of the smoothness parameter.

This choice of precision parameter is to ensure that the prior beta density has boundary values equal to zero (where the boundary values are defined either by continuity or by limits).

Hence, the higher the value of `nu.prec.inc` the more informative the prior is.

Value

An INLA model.

Examples

```
 #tryCatch version
tryCatch({
if (requireNamespace("INLA", quietly = TRUE)){
library(INLA)

# Organizing the data
data(PRprec)
data(PRborder)

Y <- rowMeans(PRprec[, 3 + 1:31])
ind <- !is.na(Y)
Y <- Y[ind]
coords <- as.matrix(PRprec[ind, 1:2])
alt <- PRprec$Altitude[ind]

seaDist <- apply(
  spDists(coords, PRborder[1034:1078, ], longlat = TRUE),
  1, min
)

# Creating INLA mesh
prdomain <- inla.nonconvex.hull(coords, -0.03, -0.05,
resolution = c(80, 80))
prmesh <- inla.mesh.2d(boundary = prdomain,
max.edge = c(0.6, 1.2), cutoff = 0.3)
```

```
# Building the A matrix
Abar <- rspde.make.A(mesh = prmesh, loc = coords)

# Building the index
mesh.index <- rspde.make.index(name = "field", mesh = prmesh)

# Creating the model
rspde_model <- rspde.matern(mesh = prmesh)

# INLA stack
stk.dat <- inla.stack(
  data = list(y = Y), A = list(Abar, 1), tag = "est",
  effects = list(
    c(
      mesh.index,
      list(Intercept = 1)
    ),
    list(
      long = inla.group(coords[, 1]),
      lat = inla.group(coords[, 2]),
      seaDist = inla.group(seaDist)
    )
  )
)

# INLA formula
f.s <- y ~ -1 + Intercept + f(seaDist, model = "rw1") +
  f(field, model = rspde_model)

# Fitting the model
rspde_fit <- inla(f.s,
  family = "Gamma", data = inla.stack.data(stk.dat),
  control.inla = list(int.strategy = "eb"),
  control.predictor = list(A = inla.stack.A(stk.dat)),
           inla.mode = "experimental"
)

# The result
summary(rspde_fit)
}
#stable.tryCatch
}, error = function(e){print("Could not run the example")})
```

`rSPDE.matern.loglike` *Object-based log-likelihood function for latent Gaussian fractional SPDE model using the rational approximations*

Description

This function evaluates the log-likelihood function for a Gaussian process with a Matern covariance function, that is observed under Gaussian measurement noise: $Y_i = u(s_i) + \epsilon_i$, where ϵ_i are iid mean-zero Gaussian variables. The latent model is approximated using the a rational approximation of the fractional SPDE model corresponding to the Gaussian process.

Usage

```
rSPDE.matern.loglike(
  object,
  Y,
  A,
  sigma.e,
  mu = 0,
  user_nu = NULL,
  user_kappa = NULL,
  user_sigma = NULL,
  user_m = NULL,
  pivot = TRUE
)
```

Arguments

object	The rational SPDE approximation, computed using matern.operators()
Y	The observations, either a vector or a matrix where the columns correspond to independent replicates of observations.
A	An observation matrix that links the measurement location to the finite element basis.
sigma.e	The standard deviation of the measurement noise.
mu	Expectation vector of the latent field (default = 0).
user_nu	If non-null, update the shape parameter of the covariance function.
user_kappa	If non-null, update the range parameter of the covariance function.
user_sigma	If non-null, update the standard deviation of the covariance function.
user_m	If non-null, update the order of the rational approximation, which needs to be a positive integer.
pivot	Should pivoting be used for the Cholesky decompositions? Default is TRUE

Value

The log-likelihood value.

See Also

matern.operators(), predict.CBrSPDEobj()

Examples

```
# this example illustrates how the function can be used for maximum
# likelihood estimation

set.seed(123)
# Sample a Gaussian Matern process on R using a rational approximation
nu <- 0.8
kappa <- 5
sigma <- 1
sigma.e <- 0.1
n.rep <- 10
n.obs <- 100
n.x <- 51

# create mass and stiffness matrices for a FEM discretization
x <- seq(from = 0, to = 1, length.out = n.x)
fem <- rSPDE.fem1d(x)

tau <- sqrt(gamma(nu) / (sigma^2 * kappa^(2 * nu) *
(4 * pi)^(1 / 2) * gamma(nu + 1 / 2)))

# Compute the covariance-based rational approximation
op_cov <- matern.operators(
  C = fem$C, G = fem$G, nu = nu,
  kappa = kappa, sigma = sigma, d = 1, m = 2
)

# Sample the model
u <- simulate(op_cov, n.rep)

# Create some data
obs.loc <- runif(n = n.obs, min = 0, max = 1)
A <- rSPDE.A1d(x, obs.loc)
noise <- rnorm(n.obs * n.rep)
dim(noise) <- c(n.obs, n.rep)
Y <- as.matrix(A %*% u + sigma.e * noise)

# Define the negative likelihood function for optimization
# using CBrSPDE.matern.loglike

# Notice that we are also using sigma instead of tau, so it can be compared
# to matern.loglike()
mlik_cov <- function(theta, Y, A, op_cov) {
  kappa <- exp(theta[1])
  sigma <- exp(theta[2])
  nu <- exp(theta[3])
  return(-rSPDE.matern.loglike(
    object = op_cov, Y = Y,
    A = A, user_kappa = kappa, user_sigma = sigma,
    user_nu = nu, sigma.e = exp(theta[4])
  ))
}
```

```
# The parameters can now be estimated by minimizing mlik with optim

# Choose some reasonable starting values depending on the size of the domain
theta0 <- log(c(sqrt(8), 1 / sqrt(var(c(Y))), 0.9, 0.01))

# run estimation and display the results
theta <- optim(theta0, mlik_cov,
  Y = Y, A = A, op_cov = op_cov,
  method = "L-BFGS-B"
)

print(data.frame(
  kappa = c(kappa, exp(theta$par[1])), sigma = c(sigma, exp(theta$par[2])),
  nu = c(nu, exp(theta$par[3])), sigma.e = c(sigma.e, exp(theta$par[4])),
  row.names = c("Truth", "Estimates")
))
```

`rspde.matern.precision` *Precision matrix of the covariance-based rational approximation of stationary Gaussian Matern random fields*

Description

`rspde.matern.precision` is used for computing the precision matrix of the covariance-based rational SPDE approximation of a stationary Gaussian random fields on R^d with a Matern covariance function

$$C(h) = \frac{\sigma^2}{2(\nu-1)\Gamma(\nu)}(\kappa h)^\nu K_\nu(\kappa h)$$

Usage

```
rspde.matern.precision(
  kappa,
  nu,
  tau = NULL,
  sigma = NULL,
  rspde_order,
  dim,
  fem_mesh_matrices,
  only_fractional = FALSE,
  return_block_list = FALSE,
  type_rational_approx = "chebfun"
)
```

Arguments

kappa	Range parameter of the covariance function.
nu	Shape parameter of the covariance function.
tau	Scale parameter of the covariance function. If sigma is not provided, tau should be provided.
sigma	Standard deviation of the covariance function. If tau is not provided, sigma should be provided.
rspde_order	The order of the rational approximation
dim	The dimension of the domain
fem_mesh_matrices	A list containing the FEM-related matrices. The list should contain elements c0, g1, g2, g3, etc.
only_fractional	Logical. Should only the fractional-order part of the precision matrix be returned?
return_block_list	Logical. For type = "covariance", should the block parts of the precision matrix be returned separately as a list?
type_rational_approx	Which type of rational approximation should be used? The current types are "chebfun", "brasil" or "chebfunLB".

Value

The precision matrix

Examples

```
set.seed(123)
nobs <- 101
x <- seq(from = 0, to = 1, length.out = nobs)
fem <- rSPDE.fem1d(x)
kappa <- 40
sigma <- 1
d <- 1
nu <- 2.6
tau <- sqrt(gamma(nu) / (kappa^(2 * nu) * (4 * pi)^(d / 2) *
gamma(nu + d / 2)))
op_cov <- matern.operators(
  C = fem$C, G = fem$G, nu = nu, kappa = kappa, sigma = sigma,
  d = 1, m = 2, compute_higher_order = TRUE
)
v <- t(rSPDE.A1d(x, 0.5))
c.true <- matern.covariance(abs(x - 0.5), kappa, nu, sigma)
Q <- rspde.matern.precision(
  kappa = kappa, nu = nu, tau = tau, rspde_order = 2, d = 1,
  fem_mesh_matrices = op_cov$fem_mesh_matrices
)
```

```
A <- Diagonal(nobs)
Abar <- cbind(A, A, A)
w <- rbind(v, v, v)
c.approx_cov <- (Abar) %*% solve(Q, w)

# plot the result and compare with the true Matern covariance
plot(x, matern.covariance(abs(x - 0.5), kappa, nu, sigma),
  type = "l", ylab = "C(h)",
  xlab = "h", main = "Matern covariance and rational approximations"
)
lines(x, c.approx_cov, col = 2)
```

`rspde.matern.precision.integer` *Precision matrix of stationary Gaussian Matern random fields with integer covariance exponent*

Description

`rspde.matern.precision.integer.opt` is used for computing the precision matrix of stationary Gaussian random fields on R^d with a Matern covariance function

$$C(h) = \frac{\sigma^2}{2(\nu-1)\Gamma(\nu)}(\kappa h)^\nu K_\nu(\kappa h)$$

, where $\alpha = \nu + d/2$ is a natural number.

Usage

```
rspde.matern.precision.integer(
  kappa,
  nu,
  tau = NULL,
  sigma = NULL,
  dim,
  fem_mesh_matrices
)
```

Arguments

`kappa`	Range parameter of the covariance function.
`nu`	Shape parameter of the covariance function.
`tau`	Scale parameter of the covariance function.
`sigma`	Standard deviation of the covariance function. If tau is not provided, sigma should be provided.
`dim`	The dimension of the domain
`fem_mesh_matrices`	A list containing the FEM-related matrices. The list should contain elements c0, g1, g2, g3, etc.

Value

The precision matrix

Examples

```
set.seed(123)
nobs <- 101
x <- seq(from = 0, to = 1, length.out = nobs)
fem <- rSPDE.fem1d(x)
kappa <- 40
sigma <- 1
d <- 1
nu <- 0.5
tau <- sqrt(gamma(nu) / (kappa^(2 * nu) *
(4 * pi)^(d / 2) * gamma(nu + d / 2)))
op_cov <- matern.operators(
  C = fem$C, G = fem$G, nu = nu, kappa = kappa, sigma = sigma,
  d = 1, m = 2
)
v <- t(rSPDE.A1d(x, 0.5))
c.true <- matern.covariance(abs(x - 0.5), kappa, nu, sigma)
Q <- rspde.matern.precision.integer(
  kappa = kappa, nu = nu, tau = tau, d = 1,
  fem_mesh_matrices = op_cov$fem_mesh_matrices
)
A <- Diagonal(nobs)
c.approx_cov <- A %*% solve(Q, v)

# plot the result and compare with the true Matern covariance
plot(x, matern.covariance(abs(x - 0.5), kappa, nu, sigma),
  type = "l", ylab = "C(h)",
  xlab = "h", main = "Matern covariance and rational approximations"
)
lines(x, c.approx_cov, col = 2)
```

---

rspde.matern.precision.integer.opt

Optimized precision matrix of stationary Gaussian Matern random fields with integer covariance exponent

---

Description

`rspde.matern.precision.integer.opt` is used for computing the optimized version of the precision matrix of stationary Gaussian random fields on R^d with a Matern covariance function

$$C(h) = \frac{\sigma^2}{2^{\nu-1}\Gamma(\nu)}(\kappa h)^\nu K_\nu(\kappa h),$$

where $\alpha = \nu + d/2$ is a natural number.

Usage

```
rspde.matern.precision.integer.opt(
  kappa,
  nu,
  tau,
  d,
  fem_matrices,
  graph = NULL
)
```

Arguments

kappa	Range parameter of the covariance function.
nu	Shape parameter of the covariance function.
tau	Scale parameter of the covariance function.
d	The dimension of the domain
fem_matrices	A list containing the FEM-related matrices. The list should contain elements C, G, G_2, G_3, etc.
graph	The sparsity graph of the matrices. If NULL, only a vector of the elements will be returned, if non-NULL, a sparse matrix will be returned.

Value

The precision matrix

---

rspde.matern.precision.opt

Optimized precision matrix of the covariance-based rational approximation

---

Description

`rspde.matern.precision` is used for computing the optimized version of the precision matrix of the covariance-based rational SPDE approximation of a stationary Gaussian random fields on R^d with a Matern covariance function

$$C(h) = \frac{\sigma^2}{2^{\nu-1}\Gamma(\nu)}(\kappa h)^\nu K_\nu(\kappa h).$$

Usage

```
rspde.matern.precision.opt(
  kappa,
  nu,
  tau,
  rspde_order,
```

```
  dim,
  fem_matrices,
  graph = NULL,
  sharp,
  type_rational_approx
)
```

Arguments

kappa	Range parameter of the covariance function.
nu	Shape parameter of the covariance function.
tau	Scale parameter of the covariance function.
rspde_order	The order of the rational approximation
dim	The dimension of the domain
fem_matrices	A list containing the FEM-related matrices. The list should contain elements C, G, G_2, G_3, etc.
graph	The sparsity graph of the matrices. If NULL, only a vector of the elements will be returned, if non-NULL, a sparse matrix will be returned.
sharp	The sparsity graph should have the correct sparsity (costs more to perform a sparsity analysis) or an upper bound for the sparsity?
type_rational_approx	Which type of rational approximation should be used? The current types are "chebfun", "brasil" or "chebfunLB".

Value

The precision matrix

---

rspde.mesh.project *Calculate a lattice projection to/from an* inla.mesh *for rSPDE objects*

Description

Calculate a lattice projection to/from an inla.mesh for rSPDE objects

Usage

```
rspde.mesh.project(...)

rspde.mesh.projector(
  mesh,
  nu = NULL,
  rspde_order = 2,
  loc = NULL,
  lattice = NULL,
```

```
  xlim = NULL,
  ylim = NULL,
  dims = c(100, 100),
  projection = NULL,
  ...
)

## S3 method for class 'inla.mesh'
rspde.mesh.project(
  mesh,
  loc = NULL,
  field = NULL,
  rspde_order = 2,
  nu = NULL,
  ...
)

## S3 method for class 'rspde.mesh.projector'
rspde.mesh.project(projector, field, ...)

## S3 method for class 'inla.mesh.1d'
rspde.mesh.project(mesh, loc, field = NULL, rspde_order = 2, nu = NULL, ...)
```

Arguments

`...`	Additional parameters.
`mesh`	An `inla.mesh` or `inla.mesh.1d` object.
`nu`	The smoothness parameter. If `NULL`, it will be assumed that nu was estimated.
`rspde_order`	The order of the rational approximation.
`loc`	Projection locations. Can be a matrix or a SpatialPoints or a SpatialPoints-DataFrame object.
`lattice`	An `inla.mesh.lattice` object.
`xlim`	X-axis limits for a lattice. For R2 meshes, defaults to covering the domain.
`ylim`	Y-axis limits for a lattice. For R2 meshes, defaults to covering the domain.
`dims`	Lattice dimensions.
`projection`	One of c("default", "longlat", "longsinlat", "mollweide").
`field`	Basis function weights, one per mesh basis function, describing the function to be evaluated at the projection locations.
`projector`	A `rspde.mesh.projector` object.

Details

This function is built upon the inla.mesh.project and inla.mesh.projector functions from INLA.

Value

A list with projection information for rspde.mesh.project. For rspde.mesh.projector(mesh, ...), a rspde.mesh.projector object. For rspde.mesh.project(projector, field, ...), a field projected from the mesh onto the locations given by the projector object.

`rspde.precision` *Precision matrices for* `inla_rspde` *objects*

Description

Precision matrices for rSPDE models

Calculates the precision matrix for given parameter values based on an `inla_rspde` model object.

Usage

```
rspde.precision(rspde, theta, optimized = FALSE)
```

Arguments

`rspde`	An `inla_rspde` object.
`theta`	The parameter vector. See the details in `rspde.matern()` to see the parameterizations.
`optimized`	Logical indicating if only the elements (the `x` slot) of the precision matrix should be returned.

Value

A sparse precision matrix.

Examples

```
 #tryCatch version
tryCatch({
if (requireNamespace("INLA", quietly = TRUE)){
library(INLA)

set.seed(1)
n <- 10

coords <- cbind(long = sample(1:n), lat = sample(1:n))

mesh <- inla.mesh.2d(coords, max.edge = c(20, 40))
rspde_model_int <- rspde.matern(mesh = mesh, nu = 1)

prec_int <- rspde.precision(rspde_model_int, theta = log(c(1, 3)))

rspde_model <- rspde.matern(mesh)
```

```
prec <- rspde.precision(rspde_model, theta = log(c(1, 3, 1.2)))
}
#stable.tryCatch
}, error = function(e){print("Could not run the example")})
```

`rspde.result` *rSPDE result extraction from INLA estimation results*

Description

Extract field and parameter values and distributions for an rspde effect from an inla result object.

Usage

```
rspde.result(inla, name, rspde, compute.summary = TRUE)
```

Arguments

`inla`	An `inla` object obtained from a call to `inla()`.
`name`	A character string with the name of the rSPDE effect in the inla formula.
`rspde`	The `inla_rspde` object used for the effect in the inla formula.
`compute.summary`	Should the summary be computed?

Value

Returns a list containing:

`marginals.kappa`	Marginal densities for kappa
`marginals.log.kappa`	Marginal densities for log(kappa)
`marginals.log.tau`	Marginal densities for log(tau)
`marginals.tau`	Marginal densities for tau
`marginals.values`	Marginal densities for the field values
`summary.log.kappa`	Summary statistics for log(kappa)
`summary.log.tau`	Summary statistics for log(tau)
`summary.values`	Summary statistics for the field values

If nu was estimated, then the list will also contain

`marginals.nu` Marginal densities for nu

If nu was estimated and a beta prior was used, then the list will also contain

`marginals.logit.nu`
Marginal densities for logit(nu)

`summary.logit.nu`
Marginal densities for logit(nu)

If nu was estimated and a truncated lognormal prior was used, then the list will also contain

`marginals.log.nu`
Marginal densities for log(nu)

`summary.log.nu` Marginal densities for log(nu)

If `compute.summary` is `TRUE`, then the list will also contain

`summary.kappa` Summary statistics for kappa

`summary.tau` Summary statistics for tau

If nu was estimated and `compute.summary` is `TRUE`, then the list will also contain

`summary.nu` Summary statistics for nu

Examples

```
 #tryCatch version
tryCatch({
if (requireNamespace("INLA", quietly = TRUE)){
library(INLA)

set.seed(123)

m <- 100
loc_2d_mesh <- matrix(runif(m * 2), m, 2)
mesh_2d <- inla.mesh.2d(
  loc = loc_2d_mesh,
  cutoff = 0.05,
  max.edge = c(0.1, 0.5)
)
sigma <- 0.01
range <- 0.2
nu <- 0.8
kappa <- sqrt(8 * nu) / range
op <- matern.operators(
  mesh = mesh_2d, nu = nu,
  kappa = kappa, sigma = sigma, m = 2
)
u <- simulate(op)
A <- inla.spde.make.A(
  mesh = mesh_2d,
  loc = loc_2d_mesh
)
```

```
sigma.e <- 0.1
y <- A %*% u + rnorm(m) * sigma.e
Abar <- rspde.make.A(mesh = mesh_2d, loc = loc_2d_mesh)
mesh.index <- rspde.make.index(name = "field", mesh = mesh_2d)
st.dat <- inla.stack(
  data = list(y = as.vector(y)),
  A = Abar,
  effects = mesh.index
)
rspde_model <- rspde.matern(
  mesh = mesh_2d,
  nu_upper_bound = 2
)
f <- y ~ -1 + f(field, model = rspde_model)
rspde_fit <- inla(f,
  data = inla.stack.data(st.dat),
  family = "gaussian",
  control.predictor =
    list(A = inla.stack.A(st.dat)),
           inla.mode = "experimental"
)
result <- rspde.result(rspde_fit, "field", rspde_model)
summary(result)
plot(result)
}
#stable.tryCatch
}, error = function(e){print("Could not run the example")})
```

`simulate.CBrSPDEobj` *Simulation of a fractional SPDE using the covariance-based rational SPDE approximation*

Description

The function samples a Gaussian random field based using the covariance-based rational SPDE approximation.

Usage

```
## S3 method for class 'CBrSPDEobj'
simulate(
  object,
  nsim = 1,
  seed = NULL,
  user_nu = NULL,
  user_kappa = NULL,
  user_sigma = NULL,
  user_m = NULL,
```

```
  pivot = TRUE,
  ...
)
```

Arguments

object	The covariance-based rational SPDE approximation, computed using matern.operators()
nsim	The number of simulations.
seed	An object specifying if and how the random number generator should be initialized ('seeded').
user_nu	If non-null, update the shape parameter of the covariance function.
user_kappa	If non-null, update the range parameter of the covariance function.
user_sigma	If non-null, update the standard deviation of the covariance function.
user_m	If non-null, update the order of the rational approximation, which needs to be a positive integer.
pivot	Should pivoting be used for the Cholesky decompositions? Default is TRUE
...	Currently not used.

Value

A matrix with the n samples as columns.

Examples

```
# Sample a Gaussian Matern process on R using a rational approximation
kappa <- 10
sigma <- 1
nu <- 0.8

# create mass and stiffness matrices for a FEM discretization
x <- seq(from = 0, to = 1, length.out = 101)
fem <- rSPDE.fem1d(x)

# compute rational approximation of covariance function at 0.5
tau <- sqrt(gamma(nu) / (sigma^2 * kappa^(2 * nu) *
(4 * pi)^(1 / 2) * gamma(nu + 1 / 2)))
op_cov <- matern.operators(
  C = fem$C, G = fem$G, nu = nu,
  kappa = kappa, sigma = sigma, d = 1, m = 2
)

# Sample the model and plot the result
Y <- simulate(op_cov)
plot(x, Y, type = "l", ylab = "u(x)", xlab = "x")
```

simulate.rSPDEobj *Simulation of a fractional SPDE using a rational SPDE approximation*

Description

The function samples a Gaussian random field based on a pre-computed rational SPDE approximation.

Usage

```
## S3 method for class 'rSPDEobj'
simulate(object, nsim = 1, seed = NULL, ...)
```

Arguments

object	The rational SPDE approximation, computed using fractional.operators(), matern.operators(), or spde.matern.operators().
nsim	The number of simulations.
seed	an object specifying if and how the random number generator should be initialized ('seeded').
...	Currently not used.

Value

A matrix with the n samples as columns.

See Also

simulate.CBrSPDEobj()

Examples

```
# Sample a Gaussian Matern process on R using a rational approximation
kappa <- 10
sigma <- 1
nu <- 0.8

# create mass and stiffness matrices for a FEM discretization
x <- seq(from = 0, to = 1, length.out = 101)
fem <- rSPDE.fem1d(x)

# compute rational approximation
op <- matern.operators(
  kappa = kappa, sigma = sigma,
  nu = nu, G = fem$G, C = fem$C, d = 1
)

# Sample the model and plot the result
```

```
Y <- simulate(op)
plot(x, Y, type = "l", ylab = "u(x)", xlab = "x")
```

`spde.matern.loglike` *Parameter-based log-likelihood for a latent Gaussian Matern SPDE model using a rational SPDE approximation*

Description

This function evaluates the log-likelihood function for observations of a Gaussian process defined as the solution to the SPDE

$$(\kappa(s) - \Delta)^{\beta}(\tau(s)u(s)) = W.$$

Usage

```
spde.matern.loglike(kappa, tau, nu, sigma.e, Y, G, C, A, d = 2, m = 1)
```

Arguments

`kappa`	Vector with the, possibly spatially varying, range parameter evaluated at the locations of the mesh used for the finite element discretization of the SPDE.
`tau`	Vector with the, possibly spatially varying, precision parameter evaluated at the locations of the mesh used for the finite element discretization of the SPDE.
`nu`	Shape parameter of the covariance function, related to β through the equation $\beta = (\nu + d/2)/2$.
`sigma.e`	The standard deviation of the measurement noise.
`Y`	The observations, either a vector or a matrix where the columns correspond to independent replicates of observations.
`G`	The stiffness matrix of a finite element discretization of the domain.
`C`	The mass matrix of a finite element discretization of the domain.
`A`	A matrix linking the measurement locations to the basis of the FEM approximation of the latent model.
`d`	The dimension of the domain. The default value is 2.
`m`	The order of the rational approximation, which needs to be a positive integer. The default value is 1.

Details

The observations are assumed to be generated as $Y_i = u(s_i) + \epsilon_i$, where ϵ_i are iid mean-zero Gaussian variables. The latent model is approximated using a rational approximation of the fractional SPDE model.

Value

The log-likelihood value.

See Also

matern.loglike(), rSPDE.loglike().

Examples

```
# this example illustrates how the function can be used for maximum
# likelihood estimation
set.seed(123)
# Sample a Gaussian Matern process on R using a rational approximation
sigma.e <- 0.1
n.rep <- 10
n.obs <- 100
n.x <- 51

# create mass and stiffness matrices for a FEM discretization
x <- seq(from = 0, to = 1, length.out = n.x)
fem <- rSPDE.fem1d(x)

tau <- rep(0.5, n.x)
nu <- 0.8
kappa <- rep(1, n.x)

# compute rational approximation
op <- spde.matern.operators(
  kappa = kappa, tau = tau, nu = nu,
  G = fem$G, C = fem$C, d = 1
)

# Sample the model
u <- simulate(op, n.rep)

# Create some data
obs.loc <- runif(n = n.obs, min = 0, max = 1)
A <- rSPDE.A1d(x, obs.loc)
noise <- rnorm(n.obs * n.rep)
dim(noise) <- c(n.obs, n.rep)
Y <- as.matrix(A %*% u + sigma.e * noise)

# define negative likelihood function for optimization using matern.loglike
mlik <- function(theta, Y, G, C, A) {
  return(-spde.matern.loglike(rep(exp(theta[1]), n.x),
    rep(exp(theta[2]), n.x),
    exp(theta[3]), exp(theta[4]),
    Y = Y, G = G, C = C, A = A, d = 1
  ))
}

#' #The parameters can now be estimated by minimizing mlik with optim

# Choose some reasonable starting values depending on the size of the domain
theta0 <- log(c(sqrt(8), 1 / sqrt(var(c(Y))), 0.9, 0.01))
```

```
# run estimation and display the results
theta <- optim(theta0, mlik, Y = Y, G = fem$G, C = fem$C, A = A)

print(data.frame(
  kappa = c(kappa[1], exp(theta$par[1])), tau = c(tau[1], exp(theta$par[2])),
  nu = c(nu, exp(theta$par[3])), sigma.e = c(sigma.e, exp(theta$par[4])),
  row.names = c("Truth", "Estimates")
))
```

`spde.matern.operators` *Rational approximations of non-stationary Gaussian SPDE Matern random fields*

Description

`spde.matern.operators` is used for computing a rational SPDE approximation of a Gaussian random fields on R^d defined as a solution to the SPDE

$$(\kappa(s) - \Delta)^{\beta}(\tau(s)u(s)) = W.$$

Usage

```
spde.matern.operators(kappa, tau, nu, G, C, d, m = 1)
```

Arguments

`kappa`	Vector with the, possibly spatially varying, range parameter evaluated at the locations of the mesh used for the finite element discretization of the SPDE.
`tau`	Vector with the, possibly spatially varying, precision parameter evaluated at the locations of the mesh used for the finite element discretization of the SPDE.
`nu`	Shape parameter of the covariance function, related to β through the equation $\beta = (\nu + d/2)/2$.
`G`	The stiffness matrix of a finite element discretization of the domain of interest.
`C`	The mass matrix of a finite element discretization of the domain of interest.
`d`	The dimension of the domain.
`m`	The order of the rational approximation, which needs to be a positive integer. The default value is 1.

Details

The approximation is based on a rational approximation of the fractional operator $(\kappa(s)^2 - \Delta)^{\beta}$, where $\beta = (\nu + d/2)/2$. This results in an approximate model on the form

$$P_l u(s) = P_r W,$$

where $P_j = p_j(L)$ are non-fractional operators defined in terms of polynomials p_j for $j = l, r$. The order of p_r is given by `m` and the order of p_l is $m + m_\beta$ where m_β is the integer part of β if $\beta > 1$ and $m_\beta = 1$ otherwise.

The discrete approximation can be written as $u = P_r x$ where $x \sim N(0, Q^{-1})$ and $Q = P_l^T C^{-1} P_l$. Note that the matrices P_r and Q may be be ill-conditioned for $m > 1$. In this case, the metehods in `operator.operations()` should be used for operations involving the matrices, since these methods are more numerically stable.

Value

`spde.matern.operators` returns an object of class "rSPDEobj. This object contains the quantities listed in the output of `fractional.operators()` as well as the smoothness parameter ν.

See Also

`fractional.operators()`, `spde.matern.operators()`, `matern.operators()`

Examples

```
# Sample non-stationary Matern field on R
tau <- 1
nu <- 0.8

# create mass and stiffness matrices for a FEM discretization
x <- seq(from = 0, to = 1, length.out = 101)
fem <- rSPDE.fem1d(x)

# define a non-stationary range parameter
kappa <- seq(from = 2, to = 20, length.out = length(x))

# compute rational approximation
op <- spde.matern.operators(
  kappa = kappa, tau = tau, nu = nu,
  G = fem$G, C = fem$C, d = 1
)

# sample the field
u <- simulate(op)

# plot the sample
plot(x, u, type = "l", ylab = "u(s)", xlab = "s")
```

`summary.CBrSPDEobj` *Summarise CBrSPDE objects*

Description

Summary method for class "CBrSPDEobj"

Usage

```
## S3 method for class 'CBrSPDEobj'
summary(object, ...)

## S3 method for class 'summary.CBrSPDEobj'
print(x, ...)

## S3 method for class 'CBrSPDEobj'
print(x, ...)
```

Arguments

object	an object of class "CBrSPDEobj", usually, a result of a call to matern.operators().
...	further arguments passed to or from other methods.
x	an object of class "summary.CBrSPDEobj", usually, a result of a call to summary.CBrSPDEobj().

Examples

```
# Compute the covariance-based rational approximation of a
# Gaussian process with a Matern covariance function on R
kappa <- 10
sigma <- 1
nu <- 0.8

# create mass and stiffness matrices for a FEM discretization
x <- seq(from = 0, to = 1, length.out = 101)
fem <- rSPDE.fem1d(x)

# compute rational approximation of covariance function at 0.5
tau <- sqrt(gamma(nu) / (sigma^2 * kappa^(2 * nu) *
(4 * pi)^(1 / 2) * gamma(nu + 1 / 2)))
op_cov <- matern.operators(
  C = fem$C, G = fem$G, nu = nu,
  kappa = kappa, sigma = sigma, d = 1, m = 2
)

op_cov
```

---

summary.rspde.result *Summary for posteriors of field parameters for an* inla_rspde *model from a* rspde.result *object*

Description

Summary for posteriors of rSPDE field parameters in their original scales.

Usage

```
## S3 method for class 'rspde.result'
summary(object, digits = 6, ...)
```

Arguments

object	A rspde.result object.
digits	integer, used for number formatting with signif()
...	Currently not used.

Value

Returns a data.frame containing the summary.

Examples

```
 #tryCatch version
tryCatch({
if (requireNamespace("INLA", quietly = TRUE)){
library(INLA)

set.seed(123)

m <- 100
loc_2d_mesh <- matrix(runif(m * 2), m, 2)
mesh_2d <- inla.mesh.2d(
  loc = loc_2d_mesh,
  cutoff = 0.05,
  max.edge = c(0.1, 0.5)
)
sigma <- 0.01
range <- 0.2
nu <- 0.8
kappa <- sqrt(8 * nu) / range
op <- matern.operators(
  mesh = mesh_2d, nu = nu,
  kappa = kappa, sigma = sigma, m = 2
)
u <- simulate(op)
A <- inla.spde.make.A(
  mesh = mesh_2d,
  loc = loc_2d_mesh
)
sigma.e <- 0.1
y <- A %*% u + rnorm(m) * sigma.e
Abar <- rspde.make.A(mesh = mesh_2d, loc = loc_2d_mesh)
mesh.index <- rspde.make.index(name = "field", mesh = mesh_2d)
st.dat <- inla.stack(
  data = list(y = as.vector(y)),
  A = Abar,
  effects = mesh.index
```

```
)
rspde_model <- rspde.matern(
  mesh = mesh_2d,
  nu_upper_bound = 2
)
f <- y ~ -1 + f(field, model = rspde_model)
rspde_fit <- inla(f,
  data = inla.stack.data(st.dat),
  family = "gaussian",
  control.predictor =
    list(A = inla.stack.A(st.dat)),
           inla.mode = "experimental"
)
result <- rspde.result(rspde_fit, "field", rspde_model)
summary(result)
}
#stable.tryCatch
}, error = function(e){print("Could not run the example")})
```

`summary.rSPDEobj` *Summarise rSPDE objects*

Description

Summary method for class "rSPDEobj"

Usage

```
## S3 method for class 'rSPDEobj'
summary(object, ...)

## S3 method for class 'summary.rSPDEobj'
print(x, ...)

## S3 method for class 'rSPDEobj'
print(x, ...)
```

Arguments

`object`	an object of class "rSPDEobj", usually, a result of a call to `fractional.operators()`, `matern.operators()`, or `spde.matern.operators()`.
`...`	further arguments passed to or from other methods.
`x`	an object of class "summary.rSPDEobj", usually, a result of a call to `summary.rSPDEobj()`.

`update.CBrSPDEobj` *Update parameters of CBrSPDEobj objects*

Description

Function to change the parameters of a CBrSPDEobj object

Usage

```
## S3 method for class 'CBrSPDEobj'
update(
  object,
  user_nu = NULL,
  user_kappa = NULL,
  user_sigma = NULL,
  user_m = NULL,
  compute_higher_order = object$higher_order,
  type_rational_approximation = object$type_rational_approximation,
  return_block_list = object$return_block_list,
  ...
)
```

Arguments

`object`	The covariance-based rational SPDE approximation, computed using `matern.operators()`
`user_nu`	If non-null, update the shape parameter of the covariance function.
`user_kappa`	If non-null, update the range parameter of the covariance function.
`user_sigma`	If non-null, update the standard deviation of the covariance function.
`user_m`	If non-null, update the order of the rational approximation, which needs to be a positive integer.
`compute_higher_order`	Logical. Should the higher order finite element matrices be computed?
`type_rational_approximation`	Which type of rational approximation should be used? The current types are "chebfun", "brasil" or "chebfunLB".
`return_block_list`	Logical. For `type = "covariance"`, should the block parts of the precision matrix be returned separately as a list?
`...`	Currently not used.

Value

It returns an object of class "CBrSPDEobj. This object contains the same quantities listed in the output of `matern.operators()`.

See Also

simulate.CBrSPDEobj(), matern.operators()

Examples

```
# Compute the covariance-based rational approximation of a
# Gaussian process with a Matern covariance function on R
kappa <- 10
sigma <- 1
nu <- 0.8

# create mass and stiffness matrices for a FEM discretization
x <- seq(from = 0, to = 1, length.out = 101)
fem <- rSPDE.fem1d(x)

# compute rational approximation of covariance function at 0.5
op_cov <- matern.operators(
  C = fem$C, G = fem$G, nu = nu,
  kappa = kappa, sigma = sigma, d = 1, m = 2
)
op_cov

# Update the range parameter of the model:
op_cov <- update(op_cov, user_kappa = 20)
op_cov
```

---

update.rSPDEobj *Update parameters of rSPDEobj objects*

Description

Function to change the parameters of a rSPDEobj object

Usage

```
## S3 method for class 'rSPDEobj'
update(
  object,
  user_nu = NULL,
  user_kappa = NULL,
  user_sigma = NULL,
  user_m = NULL,
  ...
)
```

Arguments

object	The operator-based rational SPDE approximation, computed using matern.operators() with type="operator"
user_nu	If non-null, update the shape parameter of the covariance function.
user_kappa	If non-null, update the range parameter of the covariance function.
user_sigma	If non-null, update the standard deviation of the covariance function.
user_m	If non-null, update the order of the rational approximation, which needs to be a positive integer.
...	Currently not used.

Value

It returns an object of class "rSPDEobj. This object contains the same quantities listed in the output of matern.operators().

See Also

simulate.rSPDEobj(), matern.operators()

Examples

```
# Compute the operator-based rational approximation of a
# Gaussian process with a Matern covariance function on R
kappa <- 10
sigma <- 1
nu <- 0.8

# create mass and stiffness matrices for a FEM discretization
x <- seq(from = 0, to = 1, length.out = 101)
fem <- rSPDE.fem1d(x)

# compute rational approximation of covariance function at 0.5
op <- matern.operators(
  C = fem$C, G = fem$G, nu = nu,
  kappa = kappa, sigma = sigma, d = 1, m = 2, type = "operator"
)
op

# Update the range parameter of the model:
op <- update(op, user_kappa = 20)
op
```

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