Package ‘frmqa’

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Title The Generalized Hyperbolic Distribution, Related Distributions and Their Applications in Finance

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Depends partitions, Rmpfr

Description A collection of R and C++ functions to work with the generalized hyperbolic distribution, related distributions and their applications in financial risk management and quantitative analysis.

License GPL (>= 2)

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`frmqa-package` *A Package for **F**inancial **R**isk Management and **Q**uantitative **A**nalysis*

Description

This package comprises of R and C++ functions which deal with issues relating to financial risk management and quantitative analysis by applying uni- and multi-variate generalized hyperbolic and related distributions. These issues are approached from both directions: analytical (i.e., deriving and programing analytic formulae) and numerical. Note that the latter appears to be the only approach currently used in practice because of the intractability of these families of distributions, which is caused by the presence of the modified Bessel function of the third kind BesselK $K_\lambda(z)$, in their density functions. In this package, the naming of special functions and their related functions (e.g., BesselK and incomplete BesselK functions) follows the convention used in the R package **gsl**.

Details

Package:	frmqa
Type:	Package
Version:	0.1-5
Date:	2012-11-2
License:	GPL (>= 2)

The package is under current development with more functions are planned to be added shortly.

`Bell` *Bell Coefficients Calculation*

Description

This function calculates Bell coefficients as described on p.134–139 of Comtet (1974) for calculating high order derivative of composite function $f(g(x))$ wrt x, say.

Usage

```
Bell(n)
```

Arguments

n	Derivative order `n > 1`.

Details

`Bell` calls the functions in package **partititions**, see http://cran.r-project.org/web/packages/partitions/partitions.pdf.

Note

Combinatorial computation intensifies as `n` increases.

Author(s)

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References

Comtet, L (1974) Advanced combinatorics: The art of finite and infinite expansions. Boston: D. Reidel Pub.

Hankin, R.K.S (2006) Additive integer partitions in R. Journal of Statistical Software, Code Snippets, ***16***.

Tran, T. T., Yee, W.T. and Tee, J.G (2012) Formulae for the Extended Laplace Integral and Their Statistical Applications. Working Paper.

See Also

`derivErfc`

Examples

```
Bell(5)
```

`besselK_inc_clo` *Exact Calculation of the Incomplete BesselK Function*

Description

Calculates upper and lower incomplete functions of the modified Bessel function of the third kind $K_\lambda(z)$, see details, using closed-form formulae.

Usage

```
besselK_inc_clo(x, z, lambda, lower = FALSE, expon.scaled = FALSE)
```

Arguments

`x`	Limit of the integration, `x > 0`.
`z`	Argument of the function, `z > 0`.
`lambda`	Order, $\lambda = (j + \frac{1}{2})$ with $j = 0, 1, 2, ...$
`lower`	Logical. If `TRUE` then the lower incomplete BesselK is calculated.
`expon.scaled`	Logical. If `TRUE` then the result is on an exponential scale.

Details

One of the integral representations of $K_\lambda(z)$ is given by

$$K_\lambda(z) = \sqrt{\frac{\pi}{2z}}\frac{1}{\Gamma(\lambda+\frac{1}{2})}\,e^{-z}\int_0^\infty e^{-\xi}\xi^{\lambda-1/2}\left(1+\frac{\xi}{2z}\right)^{\lambda-1/2}\,d\xi,$$

`besselK_inc_clo` evaluates closed-form formulae, which we derived to compute this integral, in the $(0,x)$ and (x,∞) intervals for the so-called lower and upper incomplete Bessel function respectively. “Exact" evaluation of the integral in these intervals can also be obtained by numerical integration using software such as Maple www.maple.com.

Author(s)

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References

Olver, F.W.J., Lozier, D.W., Boisver, R.F. and Clark, C.W (2010) Handbook of Mathematical Functions. New York: National Institute of Standards and Technology, and Cambridge University Press.

Watson, G.N (1931) A Treatise on the Theory of Bessel Functions and Their Applications to Physics. London: MacMillan and Co.

See Also

`besselK_app_ser`, `besselK_inc_erfc`

Examples

```
options(digits = 15)
## For x = 5, z = 8, lambda = 15/2 Maple 15 gives exact value of the
## lower incomplete Bessel function 0.997 761 151 460 5189(-4)
besselK_inc_clo(5, 8, 15/2, lower = TRUE, expon.scaled = FALSE)
## For x = 21, z = 8, lambda = 21/2 Maple 15 give exact value of the
## upper incomplete Bessel function 0.704 812 324 921 884 3938(-2)
besselK_inc_clo(21, 8, 21/2, lower = FALSE, expon.scaled = FALSE)
```

`besselK_inc_err` *Calculation of the Incomplete BesselK Functions in Terms of the Complementary Error Functions*

Description

Calculates incomplete BesselK functions by evaluating explicit expressions for the lower and upper incomplete BesselK in terms of the complementary error function by calling `CalIncLapInt`.

Usage

```
besselK_inc_err(x, z, lambda, bit, lower = FALSE)
```

Arguments

x	Argument, `x > 0`.
z	Argument, `z > 0`.
lambda	Argument, $\lambda = \pm(j + \frac{1}{2})$ with $j = 0, 1, 2, \ldots$.
lower	Logical. Lower incomplete Bessel function is calculated if `TRUE`.
bit	Precision bit. A positive integer greater or equal 100.

Details

One of the integral representations of the lower incomplete BesselK is given by

$$\widehat{K}_\lambda(z, x) = \frac{1}{(2z)^\lambda} \int_0^x e^{-\left\{z^2\,\xi^2 + 1/(4\,\xi^2)\right\}}\,\xi^{-2\lambda-1}\,d\xi,$$

which appears in the distribution function of the generalized inverse Gaussian distribution, see Barndorff-Nielsen(1977).

Note

Currently, analytical formulae for the incomplete BesselK functions are not available for any value of `lambda`.

Author(s)

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References

Barndorff-Nielsen, O. E (1977) Exponentially decreasing distributions for the logarithm of particle size. Proceedings of the Royal Society of London. Series A, ***353***, *401–419.*

Olver, F.W.J., Lozier, D.W., Boisver, R.F. and Clark, C.W (2010) Handbook of Mathematical Functions. New York: National Institute of Standards and Technology, and Cambridge University Press.

Tran, T. T., Yee, W.T. and Tee, J.G (2012) Formulae for the Extended Laplace Integral and Their Statistical Applications. Working Paper.

Watson, G.N (1931) A Treatise on the Theory of Bessel Functions and Their Applications to Physics. London: MacMillan and Co.

See Also

`besselK_inc_clo`, `gamma_inc_clo`, `pgig`

Examples

```
## Accuracy tests
x <- 2
z <- 5
lambda <- -c(1/2, 3/2)
lower <- sapply(lambda, function(w.)
```

```
  besselK_inc_err(x, z, lambda = w., 200, lower = TRUE))
upper <- sapply(lambda, function(w.)
  besselK_inc_err(x, z, lambda = w., 200, lower = FALSE))
## sum of two parts
(lower + upper)
## equals the whole function
(besselK(z, nu = lambda))
```

---

besselK_inc_ite *Approximation of the Incomplete BesselK Function Iteratively*

Description

Approximates incomplete modified Bessel function using the algorithm provided by Slevinsky and Safouhi (2010) and gives warnings when loss of accuracy or an exception occurs.

Usage

```
besselK_inc_ite(x, y, lambda, traceIBF = TRUE, epsilon = 0.95,
  nmax = 120)
```

Arguments

x	Argument, x > 0.
y	Argument, y > 0.
lambda	Order of the BesselK function.
traceIBF	Logical, tracks the approximation process if TRUE.
epsilon	Determines approximation accuracy which equals machine accuracy raised to the power of epsilon.
nmax	Integer. Maximum number of iterations required.

Details

One of the integral representations of the incomplete Bessel function is given by

$$K_\lambda(x, y) = \int_1^\infty e^{-xt-y/t}\, t^{-\lambda-1}\, dt,$$

besselK_inc_ite is the **R** version of the routine described in Slevinsky and Safouhi (2010) which encounters computional problems when x and y increase. Function besselK_inc_ite detects such problems and gives warnings when they occur.

Note

A C++ version of this function will be added in due course.

Author(s)

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References

Scott, D (2012) DistributionUtils: Distribution Utilities, R package version 0.5-1, http://CRAN.R-project.org/package=GeneralizedHyperbolic.

Slevinsky, R. M., and Safouhi, H (2010) A recursive algorithm for the G transformation and accurate computation of incomplete Bessel functions. Appl. Numer. Math., ***60*** *1411–1417.*

Kahan, W (1981) Why do we need a foating-point arithmetic standard? http://www.cs.berkeley.edu/wkahan/ieee754status/why-ieee.pdf.

Examples

```
## "Exact" evaluation are given by Maple 15
options(digits = 16)

## Gives accurate approximations
## x = 0.01, y = 4, lambda = 0, exact value = 2.225 310 761 266 4692
besselK_inc_ite(0.01, 4, 0, traceIBF = FALSE, epsilon = 0.95, nmax = 160)
## x = 1, y = 4, lambda = 2, exact value = 0.006 101 836 926 254 8540
besselK_inc_ite(1, 4, 2, traceIBF = FALSE, epsilon = 0.95, nmax = 160)

## NaN occurs
## x = 1, y = 1.5, lambda = 8, exact value = 0.010 515 920 838 551 3164
## Not run: besselK_inc_ite(1, 1.5, 8, traceIBF = TRUE, epsilon = 0.95,
  nmax = 180)
## End(Not run)

##Loss of accuracy
## x = 14.5, y = 19, lambda = 0, exact value = 9.556 185 644 444 739 5139(-16)
besselK_inc_ite(14.75, 19, 2, traceIBF = TRUE, epsilon = 0.95, nmax = 160)
## x = 17, y = 15, lambda = 1, exact value= 1.917 488 390 220 793 6555(-15)
besselK_inc_ite(17, 15, 1, traceIBF = TRUE, epsilon = 0.95, nmax = 160)
```

CalIncLapInt *Evaluation of Analytical Formulae for the Incomplete Laplace Integral*

Description

Evaluates analytical formulae for the lower and upper incomplete Laplace integral in terms of the complementary error function.

Usage

```
CalIncLapInt(lambda, a = 1, b = 1, x = 1, lower = TRUE, bit = 200)
```

Arguments

`lambda`	Order $\lambda = \pm(j + \frac{1}{2})$ with $j = 0, 1, 2, \ldots$.
`a`	Argument a > 0.
`b`	Argument b >= 0.
`x`	Limit of the incomplete Laplace integral.
`lower`	Logical. Lower incomplete Laplace integral is returned if `TRUE`.
`bit`	Precision bit. A positive integer greater or equal 100.

Details

The lower and upper extended Laplace integrals are given by

$$\widehat{L}_\lambda(x, a, b) = \int_0^x e^{-(a\xi^2 + b/\xi^2)} \xi^{-2\lambda - 1}\, d\xi,$$

and

$$\widetilde{L}_\lambda(x, a, b) = \int_x^\infty e^{-(a\xi^2 + b/\xi^2)} \xi^{-2\lambda - 1}\, d\xi$$

respectively. Calculation is performed using multiple precision floating-point reliably or MPFR-numbers instead of the default floating-point number in **R**, which ensure accuracy is at least 100 bit.

Author(s)

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References

*Hankin, R.K.S (2006) Additive integer partitions in R. Journal of Statistical Software, Code Snippets, **16**.*

Maechler, M Rmpfr: R MPFR - Multiple Precision Floating-Point Reliable. R package version 0.5-0, `http://CRAN.R-project.org/package=Rmpfr`.

Olver, F.W.J., Lozier, D.W., Boisver, R.F. and Clark, C.W (2010) Handbook of Mathematical Functions. New York: National Institute of Standards and Technology, and Cambridge University Press.

Tran, T. T., Yee, W.T. and Tee, J.G (2012) Formulae for the Extended Laplace Integral and Their Statistical Applications. Working Paper.

Watson, G.N (1931) A Treatise on the Theory of Bessel Functions and Their Applications to Physics. London: MacMillan and Co.

See Also

`besselK_inc_err`, `gamma_inc_err`, `pgig` derivErfc *Calculate High Order Derivatives of Composite Functions*

Description

Symbolically calculates high order derivative of composite functions involving complementary error function and exponential function.

Note

A C++ version of this function will be added in due course. It involves very intensive combinatorial computation and is not supposed to be called by the user.

Author(s)

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References

Comtet, L (1974) Advanced combinatorics: The art of finite and infinite expansions. Boston: D. Reidel Pub.

Hankin, R.K.S (2006) Additive integer partitions in R. Journal of Statistical Software, Code Snippets, ***16****.*

Olver, F.W.J., Lozier, D.W., Boisver, R.F., and Clark, C.W (2010) Handbook of Mathematical Functions. New York: National Institute of Standards and Technology, and Cambridge University Press.

gamma_inc_err *Accurate Calculation of the Incomplete Gamma Functions Using Analytical Formulae*

Description

Evaluates explicit formulae for the lower and upper incomplete gamma functions in terms of complementary error function by calling `CalIncLapInt`.

Usage

```
gamma_inc_err(x, lambda, bit, lower = FALSE)
```

Arguments

x	Argument, `x > 0`.
lambda	Argument, $\lambda = \pm(j + \frac{1}{2})$ with $j = 0, 1, 2, \ldots$.
lower	Logical. Lower incomplete gamma function is calculated if `TRUE`.
bit	Precision bit. A positive integer greater or equal 100.

Details

The lower incomplete gamma function is given by

$$\gamma(x, \lambda) = \int_0^x e^{-t}\, t^{\lambda-1}\, dt.$$

Note

This function evaluates formulae in terms of complementary error function for $\gamma(x, \lambda)$ and its upper counterpart when $\lambda = \pm(j + \frac{1}{2})$. Currently, such formulae are only available when $\lambda = \pm\frac{1}{2}$.

Author(s)

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References

Olver, F.W.J., Lozier, D.W., Boisver, R.F. and Clark, C.W (2010) Handbook of Mathematical Functions. New York: National Institute of Standards and Technology, and Cambridge University Press.

Tran, T. T (2011) Some Problems Concerning the Generalized Hyperbolic and Related Distributions. Ph.D Thesis. The University of Auckland, New Zealand.

Tran, T. T., Yee, W.T. and Tee, J.G (2012) Formulae for the Extended Laplace Integral and Their Statistical Applications. Working Paper.

Watson, G.N (1931) A Treatise on the Theory of Bessel Functions and Their Applications to Physics. London: MacMillan and Co.

See Also

CalIncLapInt, besselK_inc_clo, pgig

Examples

```
 ## Accuracy tests
 x <- 3
 lambda <- 3/2
 lower <- sapply(lambda, function(w.)
   gamma_inc_err(x, lambda = w., 200, lower = TRUE))
 upper <- sapply(lambda, function(w.)
   gamma_inc_err(x, lambda = w., 200, lower = FALSE))
## sum of two parts
 (lower + upper)
## equals the whole function
 (gamma(lambda))
```

pgig — *Accurate Evaluation of Tail Probabilities of the Generalized Inverse Gaussian Distribution*

Description

Evaluates analytical formulae for distribution functions of the generalized inverse Gaussian distribution (GIG) by calling function `besselK_inc_err`.

Usage

```
pgig(q, lambda, chi, psi, lower.tail = TRUE, bit = 200)
```

Arguments

`q`	Quantile, $q > 0$.
`lambda`	Parameter, $\lambda = \pm(j + \frac{1}{2})$ with $j = 0, 1, 2, \ldots$.
`chi`	Parameter chi > 0.
`psi`	Parameter psi > 0.
`lower.tail`	Logical. $P(W < w)$ is returned if `TRUE`.
`bit`	Precision bit. A positive integer greater or equal 100.

Details

The GIG is given by

$$GIG(w|\lambda, \chi, \psi) = \frac{(\psi/\chi)^{\lambda/2}}{2K_\lambda(\sqrt{\chi\psi})}\, e^{-\left(\chi w^{-1} + \psi w\right)/2}\, w^{\lambda-1} \qquad w > 0.$$

This distribution has been used in hydrology, reliability analysis, extreme events modelling in financial risk management, and as the mixing distribution to form the family of generalized hyperbolic distributions in statistics.

Note

This function allows for accurate evaluation of distribution functions (c.d.f and c.c.d.f) of the family of GIG distributions with $\lambda = \pm(j + \frac{1}{2})$. Currently, only c.d.f of inverse Gaussian distribution, $\lambda = -\frac{1}{2}$, is available.

Author(s)

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References

Barndorff-Nielsen, O. E (1977) Exponentially decreasing distributions for the logarithm of particle size. Proceedings of the Royal Society of London. Series A, ***353***, *401–419.*

Olver, F.W.J., Lozier, D.W., Boisver, R.F. and Clark, C.W (2010) Handbook of Mathematical Functions. New York: National Institute of Standards and Technology, and Cambridge University Press.

Tran, T. T., Yee, W.T. and Tee, J.G (2012) Formulae for the Extended Laplace Integral and Their Statistical Applications. Working Paper.

Watson, G.N (1931) A Treatise on the Theory of Bessel Functions and Their Applications to Physics. London: MacMillan and Co.

See Also

`CalIncLapInt`, `gamma_inc_err`

Examples

```
## Accuracy tests
q <- 1
chi <- 3
psi <- 15
lambda <- 5/2
lowerTail <- sapply(lambda, function(w.)
  pgig(q, chi, psi, lambda = w., lower.tail = TRUE, 200))
upperTail <- sapply(lambda, function(w.)
  pgig(q, chi, psi, lambda = w., lower.tail = FALSE, 200))
## sum of two parts equals 1
(lowerTail + upperTail)
```

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